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INVESTIGATIONS OF FREE SPACE VIBRATIONS OF A WOODWORKING SHAPER, CONSIDERED AS A MECHANICAL SYSTEM WITH THREE MAIN BODIES

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ABSTRACT

Investigation of free undamped spatial vibrations of a woodworking shaper, considered as a mechanical system with three main bodies, is the object of the proposed study. First, an original mechanic-mathematical model of a woodworking shaper developed by the authors is presented. The model considers woodworking shapers with lower placement of the spindle. In this model the woodworking shaper, the spindle and the electric motor’s rotor are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor. The model takes into account the needed mass, inertia and elastic properties of the elements of the considered system. It includes all necessary geometric parameters of this system. After that a system of matrix differential equations is compiled and analytical solutions are derived. Numerical calculations are carried out by using the developed model and modern computer programs. The calculations use the parameters of a machine used in practice. As a result of the whole study, the natural frequencies and the mode shapes of the free spatial vibrations of the studied mechanical system are obtained and illustrated.

Key words: woodworking shapers, free vibrations

1. INTRODUCTION

The tendency to significant reduction of the level of vibrations and the noise accompanying the work of modern woodworking machines in recent years implies expanding and deepening the research of the dynamic processes in these machines (Barcík, Kvietková, Aláč, 2011; Beljo-Lučić, Goglia, 2001; Trposki et all, 2013).

Woodworking shapers are in the group of woodworking machines with high levels of vibration and noise (Vukov et all, 2016). The introduction of measures for reduction of the level of vibration and noise requires understanding the essence of the phenomena typical for this machine and its individual elements (Kminiak, Siklienka, Šustek, 2016; Koljozov et all, 2016; Orlowski, Sandak, Tanaka, 2007). It is necessary to conduct concrete studies in which the machine can be considered as a mechanical vibrating system with known characteristics (Amirouche 2006, Angelov, Slavov, 2010, Coutinho 2001). Some recommendations, based on the results of these investigations and applicable to the design, manufacture and operation of woodworking shapers, can be made.

It is very important to put suitable vibration isolators between the machine and the floor (Veits, Kochura, Martinenko, 1971; Wittenburg 1977). The aim is to limit the distribution of the machine’s vibrations to the environment. Unified elastic elements with known dynamic characteristics are commonly used. The selection of vibration isolators with optimal qualities is associated with conducting a preliminary study of the vibration behaviour of the machine to assess the influence of the different elasticity coefficient of the vibration isolators.

The drive electric motor is a major element of the cutting mechanism. It is one of the sources of vibrations during operation of the woodworking machines (Stevens 2007). The rotor's vibrations, generated during operation are transmitted through its two bearing units and reach the machine's body. On the other hand, vibrations, generated by other elements of the machine, reach the rotor back through bearing units. In that way bearing units are very important for interaction of the electric motor and the machine's body.

Variable loads arise during the operation of the woodworking shaper on its working tool. They are transmitted to the spindle and by its two bearing units reach the machine's body. Vibrations, generated by other elements of the machine, reach the spindle and the cutter back through the bearing units.
It is clear that the characteristics (stiffness, damping properties, etc.) of the spindle bearings and the electric motor's rotor bearings as well as of the vibro-isolators between the machine and the floor, are important for the vibrations and the operation of the machine.

The idea that the woodworking shaper, its spindle and the electric motor's rotor are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor, derives from the written above. These elastic elements are four vibration isolators between the machine and the floor, two bearing units of the spindle and two bearing units of the electric motor's rotor.

The aim of this study is to investigate the free undamped spatial vibrations of the woodworking shaper, which is considered as a mechanical system with three main bodies. Therefore, first it is necessary to develop mechanic - mathematical model of the woodworking shaper, its spindle and the rotor of the driving electric motor. The model should take into account the characteristics of the woodworking shaper construction, the mass, inertia and elastic properties of its components as well as all needed geometric parameters of the system. A system of matrix differential equations is composed on the basis of this model and analytical solutions are presented. Numerical calculations are carried out by using the developed model and modern computer programs. The calculations use the parameters of a real machine. As a result of the study, the natural frequencies and the mode shapes of the studied mechanical system will be obtained and illustrated.

2. MATERIAL AND METHODS

This study examines the class of woodworking shapers with a low positioned spindle, which are often used in practice of the forestry industry (Filipov 1977, Obreshkov 1996). The analysis of their construction shows the strong influence of the spindle and the drive motor on the operation of the whole machine. Fig. 1 shows the general view of woodworking shapers. Fig. 2 shows a scheme of this type of woodworking shapers. The machine's body is marked with: *1*, *2* - the drive electric motor, *3* – the belt drive, *4* – the vibration isolators between the machine and the floor, *5* – the spindle with the bearings, *6* – wood shaper's saw.

Figure 1. *General view of woodworking shaper*

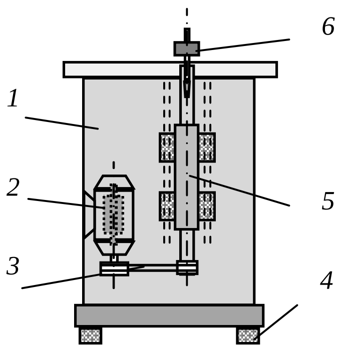

Figure 2. *Scheme of woodworking shaper*

Fig. 3 shows the spindle with its bearing units. Fig. 4 shows the spindle with fitted cutter. Fig. 5 shows the drive electric motor.

Figure 3. *Spindle with bearing units*

Figure 4. *Spindle with fitted cutter*

Figure 5. *Drive electric motor*

In the following discussions, the woodworking shaper, its spindle and the rotor of the driving electric motor are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor. These elastic elements are the four vibration isolators between the machine and the floor, two of which are the bearing units of the spindle, and the other two are the bearing units of the electric motor's rotor.

A mechanical - mathematical model of wood shapers with lower spindle is built for studying its free undamped spatial vibrations. The model is shown in Fig. 6.

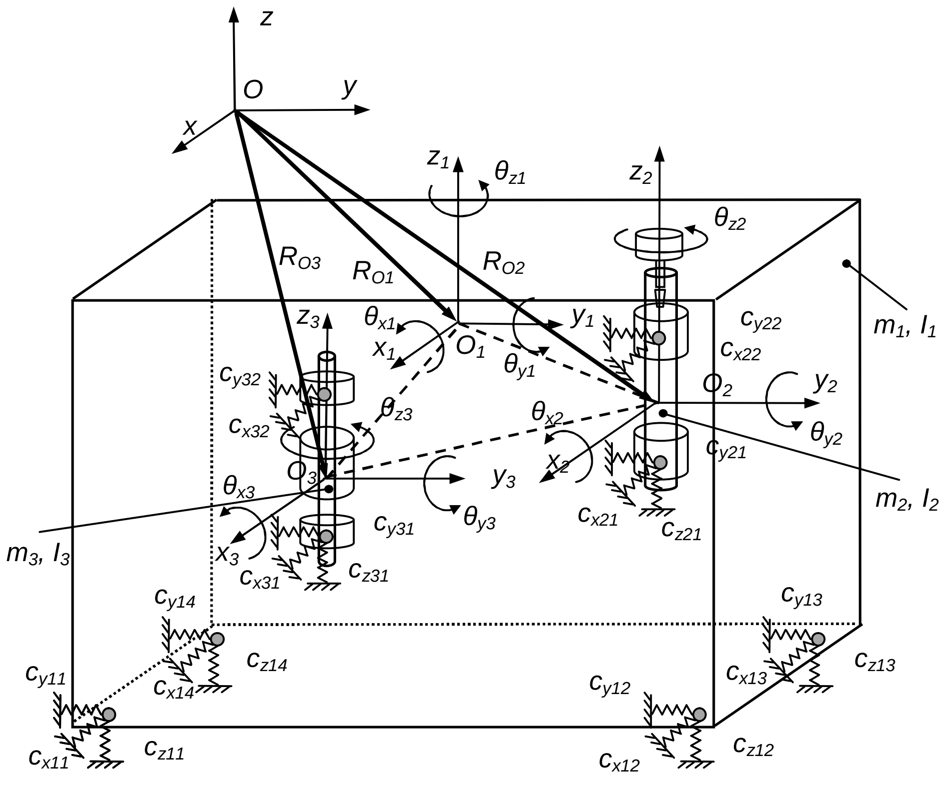

Figure 6. *Mechanic-mathematical model of the wood shaper, its spindle and motor's*

The following symbols are used:

m_1, m_2, m_3 – mass of the woodworking shaper, the spindle and the rotor of the driving electric motor;

$\mathbf{I_1}$, $\mathbf{I_2}$, $\mathbf{I_3}$ – inertia moment tensors of the woodworking shaper , the spindle and the rotor of the driving electric motor;

c_{x1i}, c_{y1i}, c_{z1i} , $i = 1, 2, 3, 4$– elastic coefficients of the vibro-isolators between the machine and the floor;
b_{x1i}, b_{y1i}, b_{z1i} , $i = 1, 2, 3, 4$– damping coefficients of the vibro-isolators between the machine and the floor;
c_{x2i}, c_{y2i}, c_{z2i} , $i = 1, 2$– elastic coefficients between the body of the machine and the spindle;
b_{x2i}, b_{y2i}, b_{z2i} , $i = 1, 2,$ – damping coefficients between the body of the machine and the spindle;
c_{x3i}, c_{y3i}, c_{z3i} , $i = 1, 2$– elastic coefficients between the body of the machine and the rotor of the driving electric motor;
b_{x3i}, b_{y3i}, b_{z3i} , $i = 1, 2,$ – damping coefficients between the body of the machine and the rotor of the driving electric motor.

The three bodies of the mechanical system perform spatial vibrations - three small translations and three small rotations relative to the axes of the rectangular local coordinate systems that are fixedly connected to the bodies.

The position of the mechanical system in space is defined by the vector of the generalized coordinates (Fig. 6), which is

$$\mathrm{q} = \left[x_1\ y_1\ z_1\ \bar{}_{x1}\ \bar{}_{y1}\ \bar{}_{z1}\ x_2\ y_2\ z_2\ \bar{}_{x_2}\ \bar{}_{y_2}\ \bar{}_{z_2}\ x_3\ y_3\ z_3\ \bar{}_{x3}\ \bar{}_{y3}\ \bar{}_{z3} \right]^{\mathrm{T}} . \tag{1}$$

The mechanical system has 18 degrees of freedom. The building of its mechanic-mathematical model is presented below.

Matrices of the transition

The matrices of transition in small vibrations from the local coordinate systems of the bodies $O_i x_i y_i z_i$ to the reference coordinate system *Oxyz* have the form

$$\mathrm{A}_{\mathrm{i}}^{0} = \begin{bmatrix} 1 & -\bar{}_{zi} & \bar{}_{yi} & x_i \\ \bar{}_{zi} & 1 & -\bar{}_{xi} & y_i \\ -\bar{}_{yi} & \bar{}_{xi} & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, i = 1,2,3 . \tag{2}$$

Vectors of position of the random points

The vector of position of the centre of mass of the relevant body, projected in the reference coordinate system, is determined with

$$\mathrm{R}_{\mathrm{Ci}}^{0} = \mathrm{A}_{\mathrm{i}}^{0} . \mathrm{r}_{\mathrm{Ci}} = \begin{bmatrix} l_{Cx} + x_i + l_{Cz} . \bar{}_{yi} - l_{Cy} . \bar{}_{zi} \\ l_{Cy} + y_i - l_{Cz} . \bar{}_{xi} + l_{Cx} . \bar{}_{zi} \\ l_{Cz} + z_i + l_{Cy} . \bar{}_{xi} - l_{Cx} . \bar{}_{yi} \\ 1 \end{bmatrix}, i = 1,2,3 , \tag{3}$$

where $\mathrm{r}_{\mathrm{Ci}} = \left[l_{Cx} \quad l_{Cy} \quad l_{Cz} \right]^{\mathrm{T}}$ is the vector of the position of the centre of mass in the local coordinate system.

Linear velocity vectors of any points

The linear velocity vector of any point *Ci* of body *i*, projected in the reference coordinate system, is obtained by differentiating by the time of the position's vector of the same point

$$\mathrm{V}_{\mathrm{Ci}}^{0}=\frac{d\mathrm{R}_{\mathrm{Ci}}^{0}}{dt}=\begin{bmatrix}\dot{x}_i+l_{Cz}\cdot\dot{-}_{yi}-l_{Cy}\cdot\dot{-}_{zi}\\ \dot{y}_i-l_{Cz}\cdot\dot{-}_{xi}+l_{Cx}\cdot\dot{-}_{zi}\\ \dot{z}_i+l_{Cy}\cdot\dot{-}_{xi}-l_{Cx}\cdot\dot{-}_{yi}\\ 0\end{bmatrix}\qquad i=1,2,3\,. \tag{4}$$

Vectors of angular velocities of the bodies

The vector of angular velocity of the body i, projected in the local coordinate system, is

$$\Omega_{\mathrm{i}}^{\mathrm{i}}=\begin{bmatrix}\dot{-}_{xi}\\ \dot{-}_{yi}\\ \dot{-}_{zi}\\ 0\end{bmatrix}\qquad i=1,2,3\,. \tag{5}$$

The kinetic energy

The kinetic energy of the mechanical system is

$$E_K=\sum_{i=1}^{3}E_{K\,i}\,, \tag{6}$$

where $E_{Ki}=\frac{1}{2}.\left(\mathrm{m}_{RR}^{i}.\mathrm{V}_{Ci}^{0\;T}.\mathrm{V}_{Ci}^{0}+\Omega_{\mathrm{i}}^{\mathrm{i}\;T}.\mathrm{I}_{\mathrm{nn}}^{i}.\Omega_{\mathrm{i}}^{\mathrm{i}}\right)$,

$$\mathrm{m}_{RR}^{i}=\int_{V_i}\rho_i\,.\mathbf{I}\,.dV_i=m_i\,.\mathrm{I}\;.$$

The elements of the matrix **M** of mass-inertial properties are defined by the expression

$$m_{i,j}=\frac{\partial^2 E_K}{\partial\dot{q}_i\,.\partial\dot{q}_j} \tag{7}$$

Potential energy

Potential energy is defined by

$$E_P=E_{PK}(q)_m+E_{PG}(q)_i \tag{8}$$

where $E_{PK}(q)_m=\sum_{m=1}^{8}\frac{1}{2}.\mathrm{q}^T.\mathrm{C}(q).\mathrm{q}$,

$E_{PG}(q)_i=\sum_{i=1}^{3}-m_i.\mathrm{g}^T.\mathrm{R}_{Ci}^{0}$,

$\mathrm{C}(q)$ is a matrix of elastic properties;

$\mathrm{g}=\begin{bmatrix}0 & 0 & g & 0\end{bmatrix}^{\mathrm{T}}$ – vector of gravitational acceleration,

m is the number of the elastic element between two bodies of the mechanical system

The elements of the matrix **C** of elastic properties are determined by the expression

$$c_{m,n}=\frac{\partial^2 E_{PK}(q)_{ij}}{\partial q_n.\partial q_m} \tag{9}$$

Differential equations

The differential equations of the free undamped spatial vibrations are derived by using the Lagrange's method. A system of differential equations which describes the small free vibrations of the examined mechanical system is obtained.

$$M.\ddot{q} + C.q = 0 \tag{10}$$

The natural frequencies and the mode shapes of the mechanical system are obtained after solving the system equations (10)

3. RESULTS AND DISCUSION

Carrying out numerical investigations of the free undamped spatial vibrations of a woodworking shaper with lower spindle requires knowledge of the parameters of its elements. Therefore the three bodies and the whole machine are modelled with software Solid Works. These models are shown respectively in Fig. 7, Fig. 8, Fig. 9 and Fig. 10. The mass centre of body 1 coincides with the centre of the local coordinate system of body 1 and the centre of the reference coordinate system. The mass centre of body 2 coincides with the centre of the local coordinate system of body 2. The mass centre of body 3 coincides with the centre of the local coordinate system of body 3.

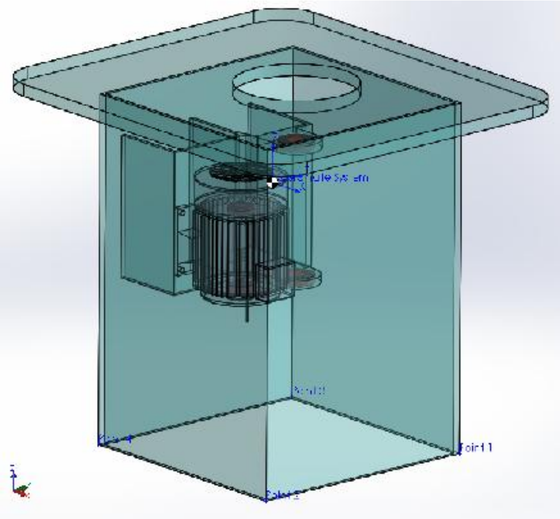

Figure 7. *Body 1*

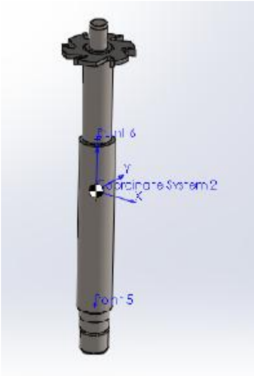

Figure 8. *Body 2*

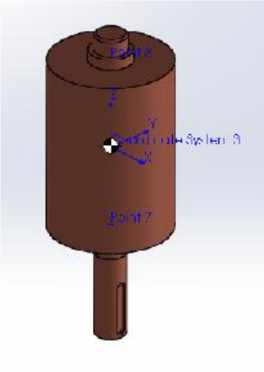

Figure 9. *Body 3*

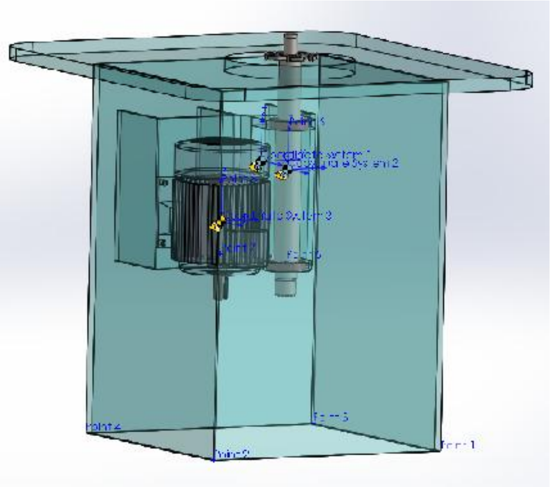

Figure 10. *Woodworking shaper*

The presented data of the machine FD-3, which is produced in ZDM – Plovdiv, is used for calculations.

Mass of the bodies: body 1 – $m_1 = 391{,}52\ kg$; body 2 – $m_2 = 11{,}123\ kg$; body 3 – $m_3 = 14{,}378\ kg$. Tensor of mass inertia moments of the body 1 to the local coordinate system of the body 1, $kg.m^2$

$$\mathrm{I}_1 = \begin{bmatrix} 49{,}2672 & -0{,}0395 & -0{,}2525 \\ -0{,}0395 & 52{,}0000 & -0{,}4405 \\ -0{,}2525 & -0{,}4405 & 47{,}9480 \end{bmatrix}.$$

Tensor of mass inertia moments of the body 2 to the local coordinate system of the body 2, $kg.m^2$

$$\mathrm{I}_2 = \begin{bmatrix} 0{,}2937 & 0 & 0 \\ 0 & 0{,}2937 & 0 \\ 0 & 0 & 0{,}0052 \end{bmatrix}.$$

Tensor of mass inertia moments of body 3 to the local coordinate system of body 3, $kg.m^2$

$$\mathrm{I}_3 = \begin{bmatrix} 0{,}0516 & 0 & 0 \\ 0 & 0{,}0516 & 0 \\ 0 & 0 & 0{,}0206 \end{bmatrix}.$$

Coordinates of the centres of mass, m

Body №	l_{Cx}	l_{Cy}	l_{Cz}
1	0	0	0
2	0,009	0,066	–0,020
3	0,019	–0,115	–0,134

Coordinates of the supporting points of the elastic elements
Coordinates in the coordinate system of body 1, *m*:

Point	l_{xi}, **m**	l_{yi}, **m**	l_{zi}, **m**
1	0,309	0,316	-0,654
2	0,309	-0,284	-0,654
3	-0,291	0,316	-0,654
4	-0,291	-0,284	-0,654
5	0,009	0,066	-0,234
6	0,009	0,066	0,076
7	0,019	-0,015	-0,210
8	0,019	-0,015	-0,050

Coordinates in the coordinate system of body 2, *m*:

Point	l_{xi}, **m**	l_{yi}, **m**	l_{zi}, **m**
5	0	0	-0,214
6	0	0	0,096

Coordinates in the coordinate system of body 3, *m*:

Point	l_{xi} , **m**	l_{yi} , **m**	l_{zi} , **m**
7	0	0	-0,076
8	0	0	0,084

Elasticity coefficients

Between Bodies	c_{xi} ,N/m	c_{yi} ,N/m	c_{zi} ,N/m
0 - 1	350000	350000	800000
1 - 2	2250000	2250000	2250000
2 - 3	2250000	2250000	2250000

Fig. 11 graphically illustrates the calculated natural frequencies [Hz] and mode shapes of free spatial vibrations of the studied mechanical system.

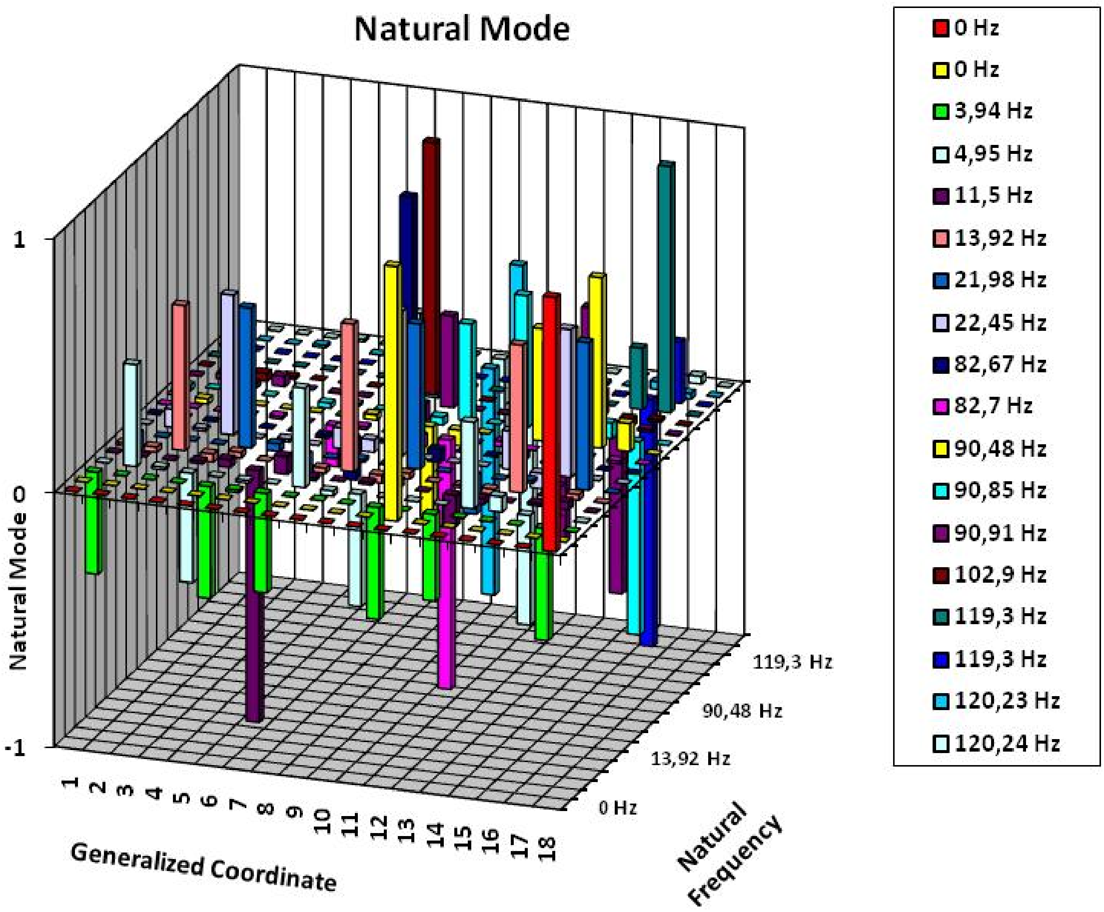

Figure 11. *Natural frequencies and mode shapes of the studied mechanical system*

Fig. 11 shows calculated natural frequencies and mode shapes of this system. Natural frequencies are 120.24 Hz; 120.23 Hz; 119.30 Hz; 119.29 Hz; 102.90 Hz; 90.91Hz; 90.85Hz; 90.48Hz; 82.70 Hz; 82.67 Hz; 22.45 Hz; 21.98 Hz; 13.92 Hz; 11.50 Hz; 4.95 Hz; 3.94 Hz ; 0 Hz; 0 Hz . These values are required for determination of the resonance zones. Knowledge of the resonance zones allows for optimizing working regimes by taking measures to avoid machine operation in these areas or to pass quickly through them. The obtained and illustrated mode shapes are useful for investigation of the vibration behaviour of the machine. Analysis of the received natural frequencies and mode shapes provides an additional opportunity for formation of reasonable recommendations for construction of these machines.

4. CONCLUSION

This study presents original investigations of the free undamped spatial vibrations of a woodworking shaper, which is considered as a mechanical system with three main bodies. An original mechanic-mathematical model of a woodworking shaper developed by the authors is presented. The model considers woodworking shapers with lower placement of the spindle. In this model the woodworking shaper, the spindle and the electric motor's rotor are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor. It takes into account the characteristics in construction of woodworking shapers. The model renders into account the needed mass, inertia and elastic properties of the elements of the considered system. It includes all necessary geometric parameters of this system. Then a compiled system of matrix differential equations is presented and analytical solutions are derived. Numerical calculations are carried out by using the developed model and modern computer programs. The calculations use the parameters of a machine, used in the practice. As a result of the whole study, the natural frequencies and mode shapes of the free spatial vibrations of the studied mechanical system are obtained and illustrated. They are necessary for determination of the resonance zones and for analyzing the vibration behaviour of the machine.

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