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SIMULATION OF SAWDUST PARTICLES IN CHAOTIC WALK USING ITS DISPERSIVE CHARACTERISTICS IN EUCLIDEAN SPACE OF THE SAWMILL ENVIRONMENT

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ABSTRACT

This paper presents the results of an investigation to study the distribution of sawdust particles in air during sawing/cutting operations in sawmill and its cluster formation and dispersion events. The main features of the proposed algorithm are: integrate the attractive features of logistic equation, Lyapunov exponent estimation and diffusive characteristic exponent in a chaotic walk in 3 different Euclidean spaces. The logistic equation control parameters ensured chaotic solution in a range, which was used to select next direction and next step size for the walk while simultaneously keeping record of distance traveled after the elapsed time step. The diffusive characteristic exponent of the average distance traveled was estimated for all parameters and linked to Lyapunov exponent to provide evidence of chaoticness or otherwise. Results yielded 9.73% of the control parameter leading to chaoticness while the super dispersive characteristic exponents for all the cases lies between 0.754 and 1.026 with the lower limit greater than 0.5 supported by literature for random walk. It was shown that super-diffusive characteristic exponent supplements and contains chaotic behavior of sawdust particles in sawmill environments. The study helps to understand the pattern of movement of inhaled sawdust particles by sawmill workers so as to remove the effects of toxicity on the body. It is an improvement on previous literature report which was limited to radius elimination method.

Key words: sawdust, toxicity, simulation, chaotic walk, sawmill, sawmill works

1. INTRODUCTION

In the last few years, there has been regular and intensive research on different aspects of sawdust utilization (Akira *et al*., 2002); (Ansari and Raofie, 2006); (Jadhav and Vanjara, 2004); (Sciban *et al*., 2006). However, relatively few articles exist on health impacts of sawdust, particularly in sawmills. Akira *et al*. (2002) used sawdust as a new base material for boilers. Ansari and Raofie (2006) used sawdust coated with polyaniline. Batzias and Sidiras (2004) used sawdust for investigation on batch and column kinetics of methylene blue and red basic 22 adsorption on $CaCl_2$. The same authors extended the study by considering prehydrolysis as a substitute to $CaCl_2$ and still used sawdust as the removal agent. Gong *et al*. (2009) used sawdust in a functionalisation arrangement by monosodium glutamate for improving its cationic sorption capacity. These studies bring an understanding of how to convert sawdust wastes into useful materials for chemical experiments. However, this line of research is quite different from the goal of the current work, which focuses on health impacts of sawdust.

More interestingly, another direction of research quantified the health hazards and abnormalities of exposures to sawdust. This has been documented as work related diseases such as asthma (Arif et al., 2003), childhood cancer in the offspring of male sawmill workers occupationally exposed to chlorophenate fungicides (Heacock *et al*., 2000), etc. Now, having considered these two groups, viz, studies concerning sawdust utilization and hazard investigations, it seems that there is a growing recognition and call for more research towards solving the health impacts of sawdust on humans than its utilization. Arising from research on health impacts of sawdust is a new and exciting research that focuses on movements of sawdust particles as it affects inhalation by sawmill workers during operation at sawmills. This is aimed at controlling sawdust toxicity and its effects on humans. Salau and Oke (2010) called for possible investigation that improves on the radius elimination method of Feder (1988) and Zmeskel *et al.* (2001), which were applied in their work. Arising from this is the need for the present investigation, which improves on the existing literature on the subject, thus bringing up a better understanding of some hidden facts about formation and dispersion of sawdust particles during operations at sawmills.

Further understanding of the problem being solved in the current work would necessitate exploring previous studies on chaotic models, which are relevant to the work. In addition, knowledge of the literature on logistic equation, Lyapunov exponent and dispersive characteristics exponents is necessary in order to properly position the current paper. Chaotic models, which may include fractal-based theoretical formulations on the estimation of chaotic behavior through fractional dimension using radius elimination method of Feder (1988) and Zmeskel *et al.* (2001), are particularly suited for simulation of formative and dispersive behaviour of sawdust particles during operation in a sawmill. Understanding this has been a useful guide in the knowledge of distribution of sawdust particles inhaled by sawdust workers so as to remove the effects of toxicity on their body quickly (Salau and Oke, 2010). The use of fractional dimension based on radius elimination method which has been formulated and applied in previous studies (Feder, 1988); (Zmeskal *et al*., 2001), (Salau and Oke, 2010) is with pros and cons.

Noteworthy among the pros are: the model is simple in implementation, requiring no complex or sophisticated approach in model analysis and validation exercises. Thus, very little training is required for those who will utilize the model, as independence in its implementation is most obvious. On the other side, the cons involve losing the incorporation of important model parameters which may make the model more robust since simplicity of model is desired. Also, many assumptions may be made for the model to be applicable. However, the integrated logistic -Lyapunov-dispersive model is also with its pros and cons. Notable pros for the model include a strong base that incorporates many features that are missing in the fractional dimension model. One of the cons is that due to the more complex nature of the model, more time, training and skill are needed to master the implementation procedure. In the following paragraph, we present some literature information on logistic models.

The logistic model has been widely applied in stability and permanence studies of physical systems (Jiang *et al*., 2008); (Li *et al*., 2007), microbial growth (Peleg *et al*., 2007), single-species populations (Sakanoue, 2007), disaster response activities (Yi and Ozdamar, 2007), and in studying chaotic situations (Sen and Mukherjee, 2007). The literature also has an extensive documentation on the various aspects of logistics model that have been improved over years. These include extensions to logistic model by considering the error growth (Sancho, 2008), delay properties (Li, 2008), (Lisena 2008), Yang and Yuan (2008), Cui and Li (2007) and periodic nature of logistic model (Lisena, 2007). Numerical methods have also been applied to logistic equations (Afrouzi *et al*., 2007a,b), non-linear analysis has also been aused (Dong and Liu, 2007), fractional order (El-Sayed *et al*., 2007), analytical solution (Thornley *et al*., 2007) and differential equations (Yuan, 2007). Other studies that relate to this work include randomness (Aquion et al., 2001), chaotic diffusion (Kolovsky, 1997) and dynamic chaos (Shyy, 1991). Several investigations have been carried out on logistic equation with a view to capturing the behaviour of a number of functions. Afrouzi *et al*. (2007a) considered a reaction-diffusion equation,

$$\Delta u\,(x) + au\,(x)^2 - ch\,(x) = 0 \tag{1}$$

with Dirichlet boundary condition. This paper has not demonstrated the possibility of integrating the three functions (i.e. logistic equation, Lyaponor exponent and diffusive equation) to improve on the performance of the integrated model. In another paper, Afrouzi *et al*. (2007b) proposed a variant of the earlier model using a numerical method based on sub-super solution in which the previous equation is reduced to $au\text{-}bu^2$. Also, no form of integration has been proposed by Afrouzi *et al*. (2007b).

Apart from the literature documentation by Feder (1988) and Zmeskel et al.(2001), which the current work compares itself with, there is a growing literature that either studied the problem using radius disk elimination method or utilized random walk principles in the solution approach to a similar problem. The case of Salau and Oke (2010) that addressed sawdust movement problem using radius elimination method has earlier been mentioned. However, a complementary article relates to the work of Alabi *et al*.(2000) that reported correlation properties in English scientific text by means of a random walk, which is related to the principles adopted in the final phase of the model formulation in this work. Alabi *et al*. (2008) utilized random walk principles to judge the quality of engineering write ups based on the average distance of a number of random walk from a starting point. These two streams of research only complement the efforts in the literature but have not addressed the problem solved in the current work.

The primary objective of this paper is to demonstrate that, when properly integrated, the combination of logistic equation, Lyapunov exponent and the diffusive characteristics exponents can be shown to be a solution to the dispersion mode-capture problem of sawdust movement in a sawmill. The compatibility of the three algorithms, that is logistic equation with spread characteristics, the Lyapunov exponent which is taken as a function of the logistic equation, and the power low attribute of the diffusive characteristic exponent, improves the overall efficiency of the simulations, since all these models are linked to one another and the attributes shared.

The structure of the article is as follows. The introduction provided a strong motivation for the study and a justification for the study through an adequate review of the literature to demonstrate gap that is available in the literature and one that the current paper fills. Section 2 presents the material and methods, containing the mathematical framework for the study. This started with the declaration of the logistic model and progressed to show how it could be integrated with the Lyapunov exponent. Further, it showed how dispersive exponent could be integrated to the frame work. Section 3 presents the results and analysis to demonstrate the trend in the analysis. Section 4 is the discussion. In section 5, the concluding remarks are given.

2. MATERIAL AND METHODS

2.1 Problem description

The problem is formulated with due consideration to sawdust behavioral pattern, which is similar to the behavior of particles suspended in air, traveling around space in normal day-to-day activities (Fig. 1). Consider two sawdust particles *A* and *A'* that are emitted from the cutting machine during the sawing operation of logs of wood in a sawmill. These sawdust particles walk around in 1-*D*, 2-*D* and 3-*D* Euclidean spaces according to any or a combination of walk patterns such as random, stochastic, chaotic. Thus, we may have several combinations of walk behaviours, which may bring in some complex analysis. Therefore, as a research strategy, we focus on random and chaotic walk patterns so as to illustrate the methodology proposed in this article. Thus, from the sawdust particles *A* and *A'* mentioned above, particle *A* may be governed by a walk pattern described by pure randomness that obeys uniform distribution. It means that if a large sample is collected and analyzed, the result will reveal that a small sample percentage of the population would be to the left and right hand sides of the normal distribution curve, while a high percentage would be recorded for large samples, which fall in between these two extremes. This walk pattern of particle *A* is different from that of sawdust particle *A'*.

For the latter, the walk pattern is governed by chaotic distribution, which may be obtainable from the solution of logistic equation. Notice that sawdust particles *A* and *A'* are alike in all respect except that the walks that drive them are different. The objective is to evaluate them for walk performance between two points at known distance apart. Thus, the walks have been established to be driven by distributions (uniform and chaotic). A major difference between sawdust particles *A* and *A'* is the driving force, determined by the distribution that propels them in the Euclidean spaces. The exponent in this article reveals how fast large number of sawdust particles *A* and *A'* are spreading around in Euclidean space with elapse of time. The higher the exponent, the faster the spread rate and vice-versa. This spread action is analogical to the dispersion concept from which the study obtains a keyword utilized in its title. Note that it takes time for bad odor to spread through space. Similarly, the walk pattern of the sawdust is governed by the same principle. Lyapunov exponent is a mathematical tool for knowing whether or not a set of solutions obtained from logistic model is chaotic. The test is essential as a driving force for sawdust particle *A*'.

Figure 1. *Sawdust particles, source (wood smoothening machine) and the sawmill environment*

2.2 Assumptions

There is a number of assumptions guiding the use of the model. The first assumption relates to the weather conditions. The motion of sawdust particles is obviously affected by humidity in the surrounding environment where the sawmill is situated. The speed of movement of sawdust particles is also affected by the direction and intensity of wind. If wind blows in direction opposite to the direction of the initial movement of the affected particles, the particle speed is retarded and may consequently change direction to the opposite or some other directions depending on the magnitude and direction of the wind. The second assumption relates to the dryness or otherwise of the timber to be sawn. A dried timber produces sawdust that is emitted at high speed from the high feed and cutting rates of the band saw. If the timber is relatively wet, the amount and sizes of sawdust particles are affected; more weighty sawdust particles will be sawn. The third assumption relates to the conditions of the cutting tools. Blunt tools produce less cutting force and consequently less quantity of sawdust may be produced. In practice, although produced from similar shapes, the sawdust particles are not perfectly alike. Also their surface areas and weights vary. However, for ease of modelling and computation, these shapes are taken to be the same, which is the fourth assumption. The environment that the study is assumed to have carried out is the dry season in Nigeria. It is acknowledged that different observations in results may be obtained when condition of very cold (snow period) season such as in cold regions of Scotland and Canada, among others, is considered. The depth of cut of timber, cutting speed, type and the amount of lubricants applied during the wood cutting process are strongly influenced by the skill and experience of the machine operator. Thus, the fifth assumption is that an operator of average experience should be hired so that the deviation of the results from normal would be minimal.

2.3 Mathematical framework

The framework upon which the current paper is built is the integration of three models: Logistic equation (LE), the Lyapunov exponent and the diffusive characterizing exponent (DC). The integrated model may be conveniently referred to as the LE-LY-DC model and has shared attributes of the respective components. The formulation of the model, which starts from the logistic equation framework, could be used to understand the spread of sawdust particles dispersed from the machine source into the sawmill environment. Borrowed from the literature, the expression for the logistic equation relevant to the current study includes:

$$X_{n+1} = K * X_n * (1.0 - X_n) \tag{2}$$

where X_{n+1} is the normalized population of workers in the sawmill who are exposed to the sawdust dispersion effect at time n+1. X_n is the normalized population of workers in the sawmill who are exposed to the sawdust dispersion effect at time *n*. *K* is the level of spread control put in place. For *K*, for instance, the proper ventilation may discourage excessive inhalation by the sawdust worker. Others may utilize fan to blow the spreading sawdust. If *K* is high, it implies that the control is effective and desirable. *K* may therefore be calculated based on the level of sophistication of control put in place in terms of protection of human exposed parts to the inhalation of sawdust. Eye, nose, mouth and skin are contact areas. Gadgets meant for the protection of these body parts increase the level of control. Thus, *K* may be evaluated in an increasing function and for practical purposes it is taken as being varied between 0.001 and 3.999 in constant step of 0.001. *K* is an indicator of control strength of the sawdust particle. For value of *K* outside the range 0 to 3.999, the solution of logistic model equation (3) will grow without bound and this is totally unwanted in the current study. The solution is expected to be bouncing around within around definite limits. Specifically the solution is to be bound between minimum of zero and maximum of 1. However, the distribution of the solution can be fixed, periodic or chaotic behaviour, but never random behaviour. The chaotic solution is what the present study utilizes to drive the sawdust particle that is marked *A*' in the Euclidean space. For the same reason, we employed Lyapunov tool to make a pass or fail test for the solution generated for specified *K*-value. A pass implies the solutions obtained at *K*-value specified is chaotic and fail, if otherwise. The constant step of 0.001 is to ensure thorough check for all *K*-values between the 0 and 3.999 limits that lead to chaotic solution.

Since the sawmill is assumed to be in operation and the sawmill workers must have worked for a period of time before this study is carried out to test the system parameters, the initial normalized population of sawmill workers affected by the sawdust dispersion and inhalation problem, X_0 used for all cases studied is 0.3. This is similar to the fractional man problem in manufacturing studies. The interpretation is that if it is assumed that a 100% affected population shows signs of reaction to the side effects of sawdust inhalation, 33.33% progress in inhalation has been made and with 66.70% additional period with constant level of activities in the sawmill, the current population would be matured to show signs of reaction or side effects.

Recall that having defined equation (2) for the logistic equation, the next step is to define equations for the Lyapunov exponent and the dispersion characterizing exponent and show how they are integrated. However, before doing this, we define variants of equation (2) as defined by other authors so that comparison could be made for the purpose of validating the model. These are defined in the next set of models. The first variant of the logistic model was proposed by Afrouzi *et al*. (2007a), who defined the logistic model as:

$$X_{n+1}= K^* (1.0-X_n) \tag{3}$$

Notice that in comparison with our model stated in equation (2), X_n, which is a second coefficient of the terms in bracket eliminated. The definitions for X_{n+1} , K and X_n still holds as for the previous definitions in equation (2). We further progressed by comparing the second variant of equation (2) with the original equation (2) established above. This second variant is stated by Afrouzi *et al*. (2007a) from the previous work quoted as:

$$X_{n+1} = \Delta K * X_{\mathrm{n}} + 1.0 * K * X_n + 1.0 * K * X_n^2 - 1.0 * X_n \tag{4}$$

With the same definition of terms as stated earlier, the additional term not earlier defined, ΔK, means an increment in the value of K, i.e. 0.001. There is a third variant of the logistic model, which was developed by Afrouzi *et al*. (2007a), which is stated as

$$X_{n+1}= K^*X_n + K^*X_n + K^* X_n^2 \tag{5}$$

All the terms in equation (5) have been defined previously and are still the same. Furthermore, there is a fourth variant of the logistic model, as defined by Afronzi *et al*. (2007b). The framework is stated as:

$$X_{n+1}= K^*X^n - \mathrm{K}^* X_n^2 \tag{6}$$

The terms in equation (6) have also been defined previously. It should be re-emphasized that equation (2) is utilized in the work while equations (3) to (6) are variously combined with the LY-DC model combinations to obtain validity results with which comparison of obtained results could be made. Now, there is a need to progress on other equations which would be integrated with the logistic equations defined earlier. The Lyapunov exponent estimate is expressed in equation (7) and is used to estimate the Lyapunov exponent of logistic equation. Thus, irrespective of the form of the equation listed in equations (2) to (6), there is a common expression for X_{n+1}, which is the input for the Lyapunov exponent, stated as

$$\text{Lyapunov exponent} = \lambda = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N} \log_2 \left| \frac{df(X_{n+1})}{dx} \right| \tag{7}$$

Later, some study cases were established where N was bounded at 2000 to ensure reliability of the estimated Lyapunov estimate. The next advancement entails mathematically defining the behavior of the sawdust particles in Euclidean space, bearing in mind the possible chaotic walk of the sawdust particles and its dispersion characteristics. It thus means that parameters need to be defined and related to average distance moved by the sawdust particles from the woodworking machine that produces it. We then have equation (8):

$$D_{av} \alpha T^{\beta} \tag{8}$$

Where D_{av} represents the average distance traveled away from the machine source of sawdust particles (datum) after a known elapsed time step, T. β refers to the dispersion characterizing exponent. It should be noted that 200 walks are accounted for in the study and the elapsed time step varies from 1 to 2000 in constant increment of 1.

3. RESULTS

A Fortran program was developed, tested and implemented for the model developed in equations (2), (7) and (8) and the program was run on a Pentium based computer with a reasonable speed. The program was developed based on each of the 1-Dimension,2-Dimension and 3-Dimension Euclidean space investigation. The fourth platform was the shortest and it enables the results for the generation of figures 3 and 4 that are presented in the results section. The flow chart for the solution procedure (Fig. 1a, 1b and 2) shows a linkage of the logistic model, the Lyapunov exponent and the dispersion characterizing exponent. In developing codes for the logistic model, its conceptualization as a dynamic system was made. As a dynamic system, it exhibits three levels of solutions: initial, transition and steady. This is similar to the dynamic attribute of an aero-plane with the same three levels representing a run-off, taking off into air and stabilizing for a very long journey. Thus, referring to the flow chart (Fig.1a), the content of the second rectangular box represents the initial solution.

For the transition solutions, the content of the fourth rectangular box (Fig. 1a) reflects this. Inside this box, the logistic model is solved iteratively in "1tr"times. The content of the sixth rectangular box (Fig.1a) presents this iteratively until (J=Nstep) condition is met. This represents the second appearance of the logistic model. Lyapunov exponent is computed iteratively in this box through the seventh rectangular box. The final average Lyapunov exponent is computed in the first rectangular box in Fig. 1b. Notice that the third rectangular box in figure 1b is for initiation of chaotic walks depending on whether we are investigating on 1-D, 2-D or 3-D. The one indicated on the flow chart is specifically for two dimensions (2-D). The fourth rectangle in Fig. 1b computes more steady solutions of logistic model iteratively and uses the results constructively for chaotic walks in 1-D, 2-D or 3-D. as the case may be. The fifth rectangle in fig. 1b calls the subroutine in Fig. 2 for Dm and Cm computation, which represents the characterizing exponent while Cm is the intercept. Finally, the results of the investigation are written into files 2 and 3 according to format (65). Notice that the procedure is repeated for the next KKK until KKK=Nend. In the nomenclature are defined the meanings of the various terms utilized in the program.

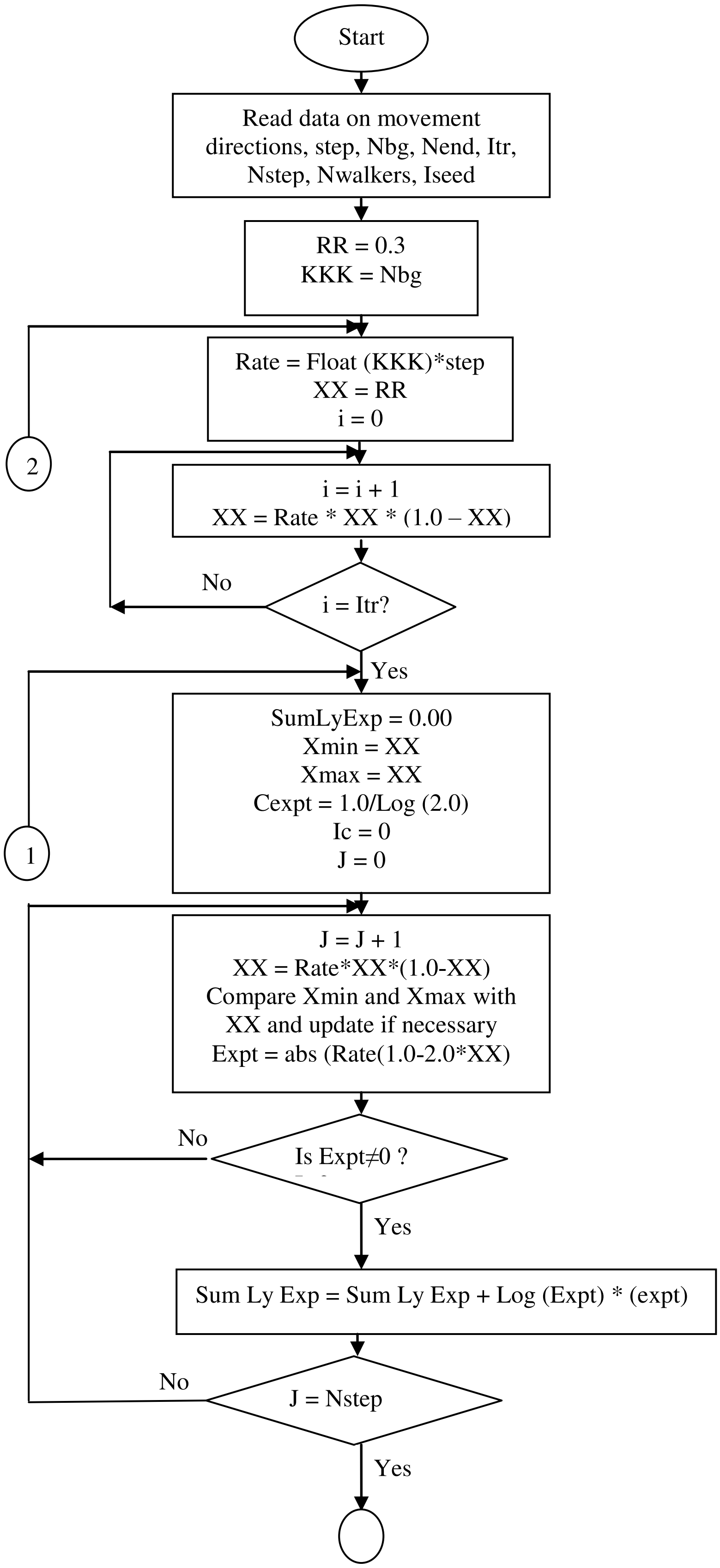

Figure 1a. *Flow chart of computation of model results*

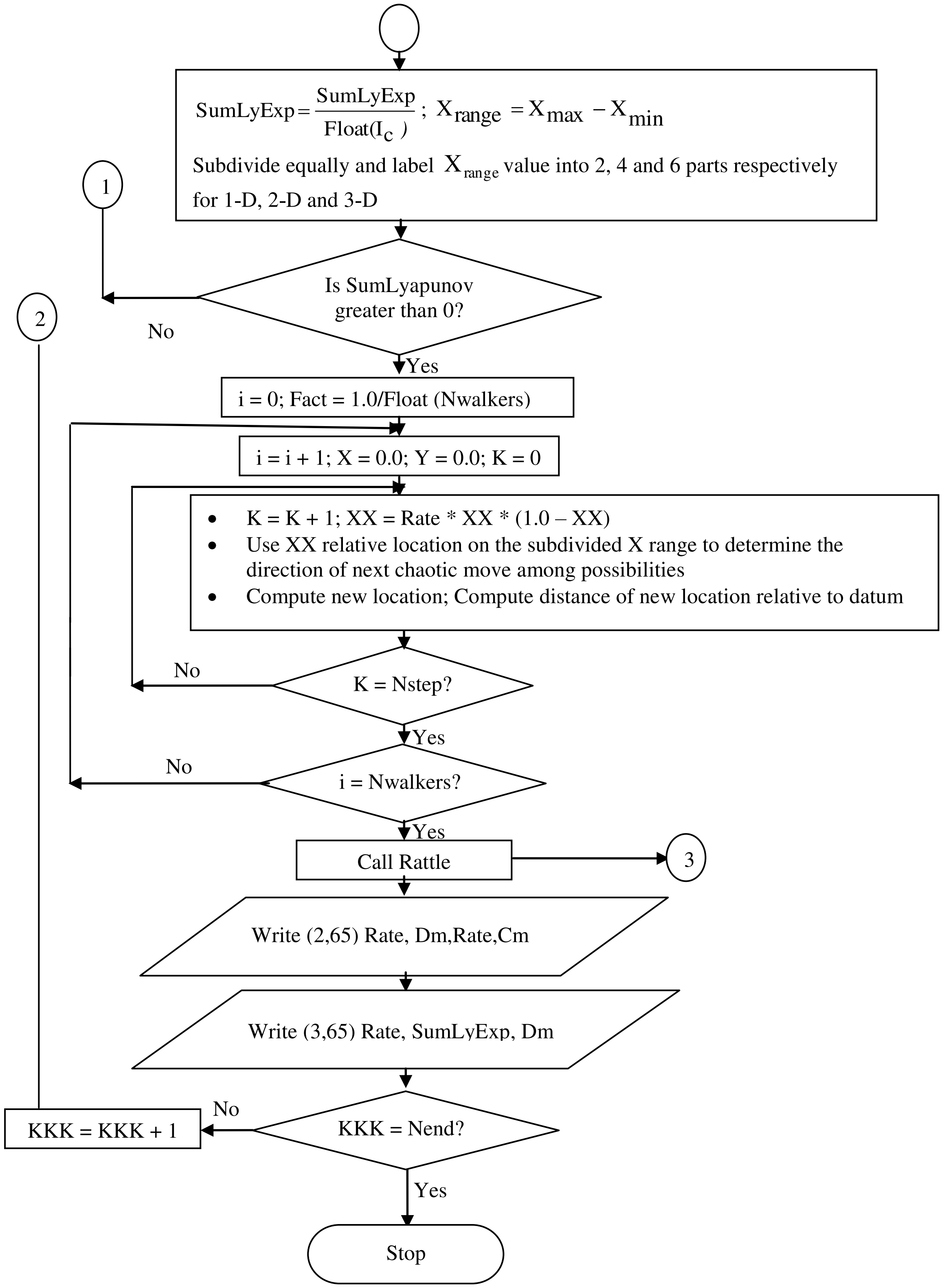

Figure 1b. *Flow chart of computation of model results (extended part)*

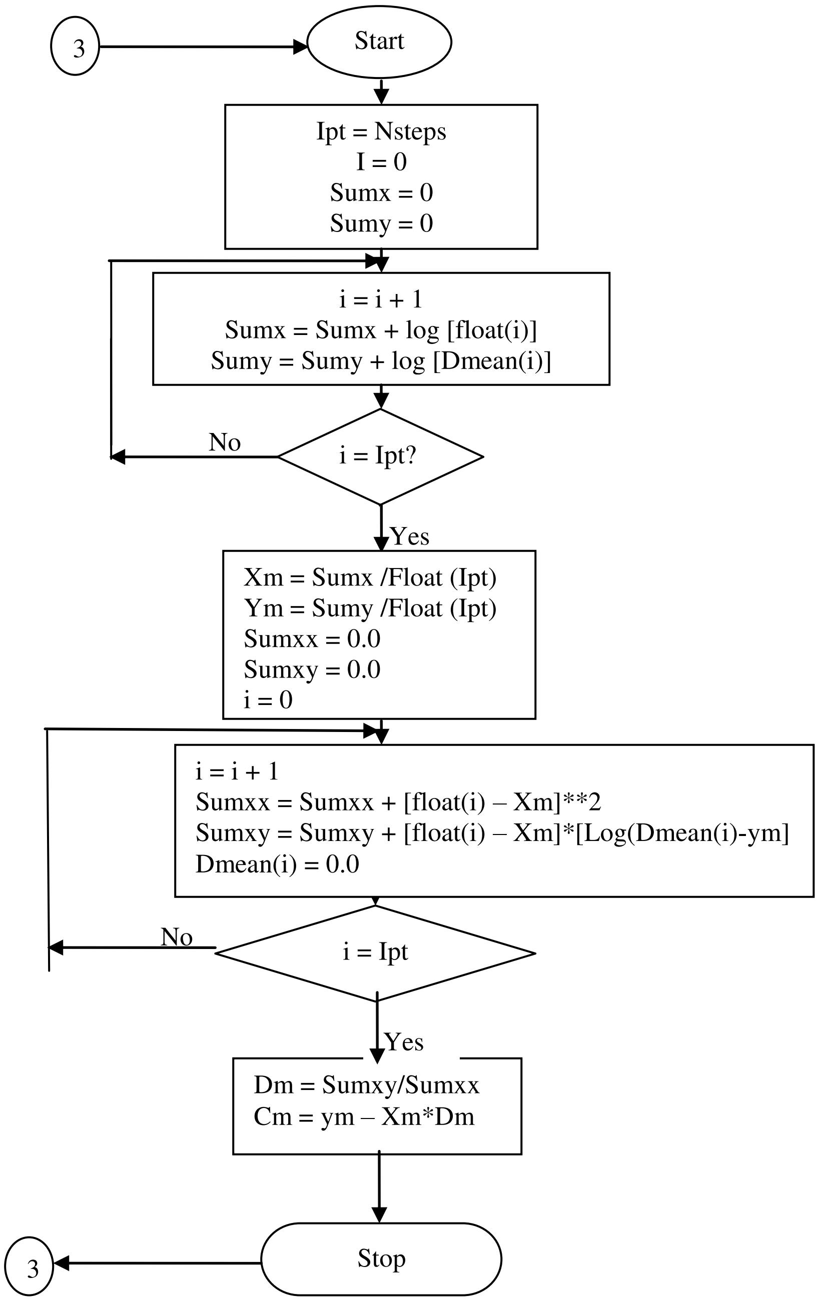

Figure 2. *Flow chart of subroutine Ratte*

For the solutions described for *K* in section 2.3, the choice of directions is governed by:

1-D Euclidean Space:
(1) Move a step (size picked randomly from 0 to 1.0) to the *right* if solution is $0 \le s \le 0.5$
(2) Move a step (size picked randomly from 0 to 1.0) to the *left* if solution is $0.5 \le s \le 1.0$
2-D Euclidean Space
(1) Move a step (size picked randomly from 0 to 1.0) to the *North* if solution is $0 \le s \le 0.25$
(2) Move a step (size picked randomly from 0 to 1.0) to the *South* if solution is $0.25 \le s \le 0.5$
(3) Move a step (size picked randomly from 0 to 1.0) to the *East* if solution is $0.5 \le s \le 0.75$
(4) Move a step (size picked randomly from 0 to 1.0) to the *West* if solution is $0.75 \le s \le 1.0$

3-D Euclidean Space

(1) Move a step (size picked randomly from 0 to 1.0) to the *North* if solution is $0 \le s \le 0.167$

(2) Move a step (size picked randomly from 0 to 1.0) to the *South* if solution is $0.167 \le s \le 0.334$

(3) Move a step (size picked randomly from 0 to 1.0) to the *East* if solution is $0.334 \le s \le 0.501$

(4) Move a step (size picked randomly from 0 to 1.0) to the *West* if solution is $0.501 \le s \le 0.608$

(5) Move a step (size picked randomly from 0 to 1.0) to the *Toward Roof* if solution is $0.608 \le s \le 0.775$

(6) Move a step (size picked randomly from 0 to 1.0) to the *Toward Floor* if solution is $0.775 \le s \le 1.0$

Where's' is the same as sequence of chaotic solutions (X_n) obtained solving equation (3) in the paper. The results obtained from running the program of which the flow charts were earlier displayed are presented in this section. Primarily, the behavioral pattern of 500 consecutive steps of a random walk in 2-Dimension is graphically displayed (Fig. 3).

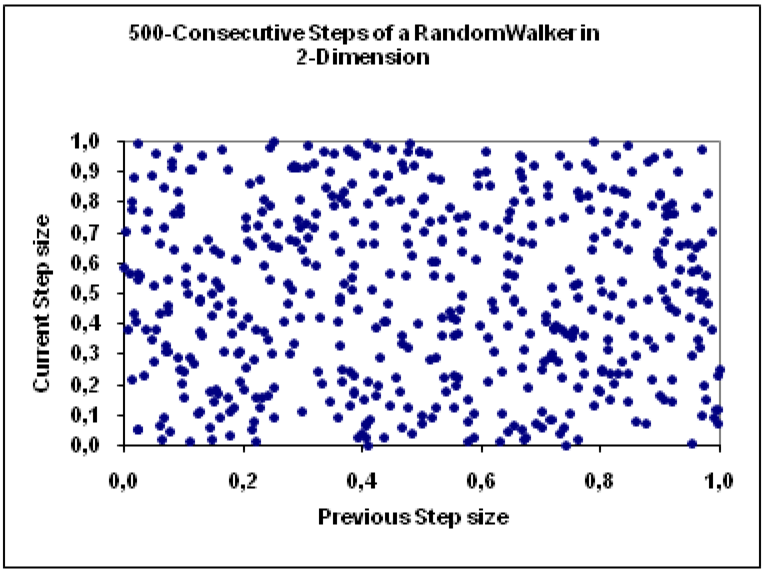

Figure 3. *500-Consecutive Steps of a Random Walker in 2-Dimension*

In Fig. 4, the sawdust particles movement behavioral pattern was demonstrated with 500 consecutive steps of a chaotic walk in 2-dimensions.

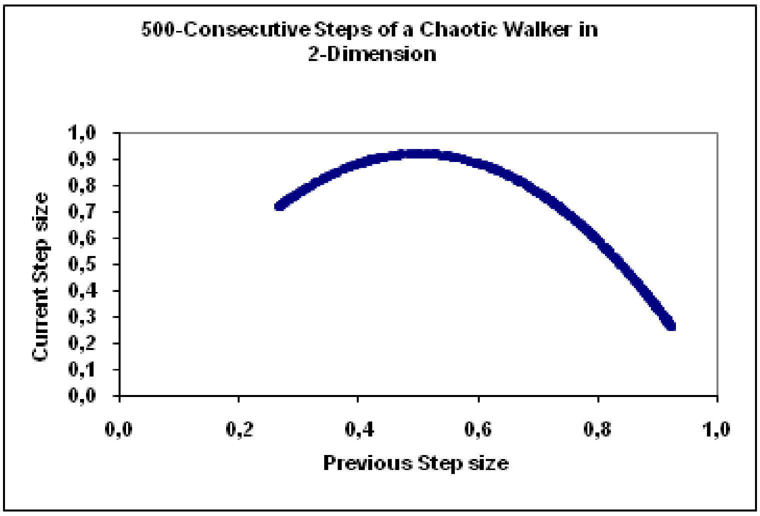

Figure 4. *500-Consecutive Steps of a Chaotic Walker in 2-Dimension*

Table 1 shows the combined information on simulation concerning control parameters, Lyapunov exponents and dispersion characteristics. It should be noted that Table 1 shows an extract of less than 10% of the sampled cases results. The remaining 90% results are not chaotic and therefore not relevant to this study. The table can serve reference purposes as literature reports on chaotic behavior of Logistic equation are often presented in graphic.

Table 1. *Control parameters, Lyapunov exponents and diffusive characteristics exponents*

Control parameters with chaotic solution	**Corresponding estimated Lyapunov exponents**	**Diffusive characteristic exponents of chaotic walkers average distance travelled reference a datum after a given elapsed time steps in 3-different Euclidean spaces**		
		1-Dimension	**2-Dimension**	**3-Dimension**
3.570	0.003	0.984	0.993	0.994
3.571	0.015	0.985	0.993	0.994
3.572	0.016	0.985	0.993	0.994
3.573	0.018	0.985	0.993	0.994
3.574	0.024	0.989	0.993	0.993
3.575	0.030	0.987	0.993	0.994
3.576	0.030	0.987	0.993	0.993
3.577	0.032	0.987	0.993	0.994
3.578	0.028	0.981	0.993	0.991
3.579	0.035	0.981	0.993	0.992
3.580	0.038	0.981	0.992	0.990
3.581	0.035	0.981	0.992	0.990
3.582	0.021	0.982	0.992	0.989
3.584	0.033	0.982	0.993	0.988
3.585	0.042	0.982	0.992	0.989
3.586	0.042	0.982	0.992	0.989
3.587	0.045	0.981	0.992	0.989
3.588	0.048	0.982	0.992	0.989
3.589	0.048	0.982	0.992	0.990
3.590	0.049	0.982	0.992	0.989
3.591	0.054	0.982	0.992	0.990
…	…	…	…	…
3.990	0.223	1.000	0.988	0.972
3.991	0.219	0.996	0.997	0.975
3.992	0.219	0.998	0.996	0.975
3.993	0.221	1.000	0.992	0.971
3.994	0.228	0.998	0.996	0.981
3.995	0.228	0.995	0.991	0.978
3.996	0.229	0.995	0.999	0.973
3.997	0.231	0.993	0.997	0.983
3.998	0.233	0.998	1.002	0.981
3.999	0.236	0.998	0.996	0.965
Range of characteristic exponents in Euclidean spaces		0.981 to 1.022	0.810 to 1.021	0.754 to 1.026

Furthermore, Fig. 5 shows the relationship between the control parameters and characteristic exponents using 1-Dimensional data. A trend line is also fitted to understand the variation of the data points from the line.

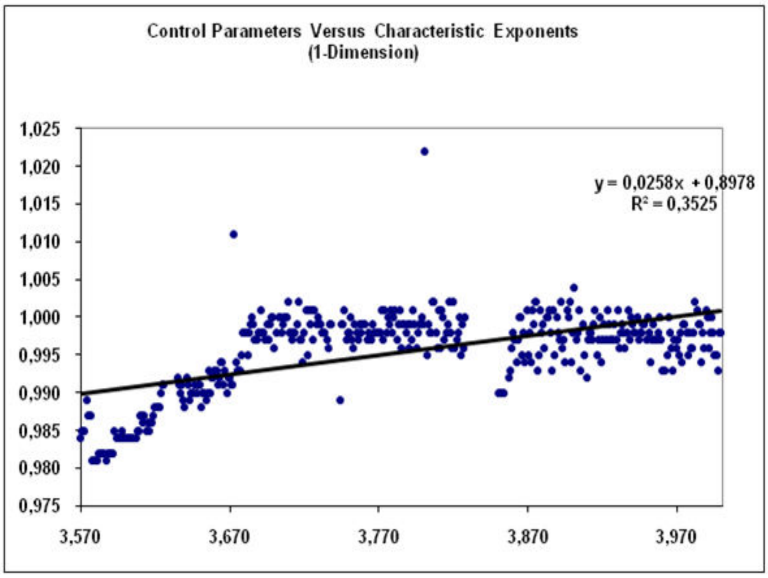

Figure 5. *Control Parameters versus Characteristic Exponents (1-Dimension)*

Fig. 6 shows the relationship between the control parameters and characteristic exponent in 2 dimensions

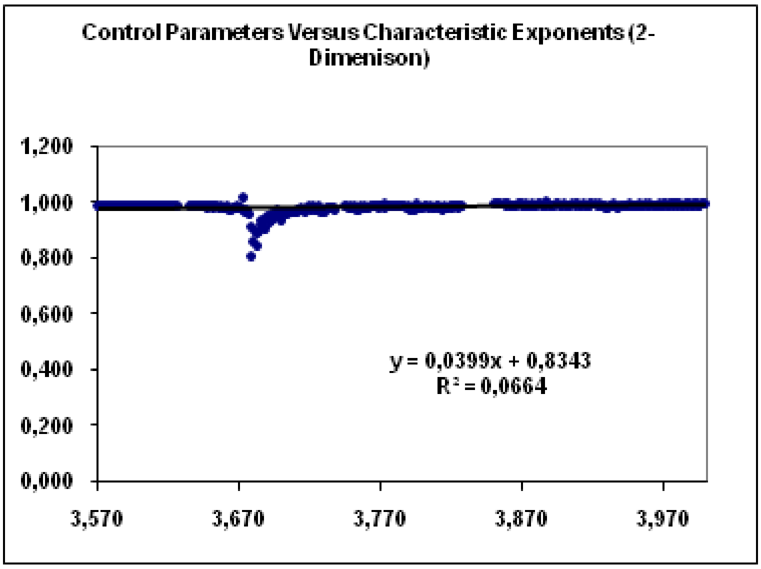

Figure 6. *Control Parameters versus Characteristic Exponents (2-Dimension)*

In Fig. 7, a relationship between control parameters and characteristic exponent in 3-dimension is sought.

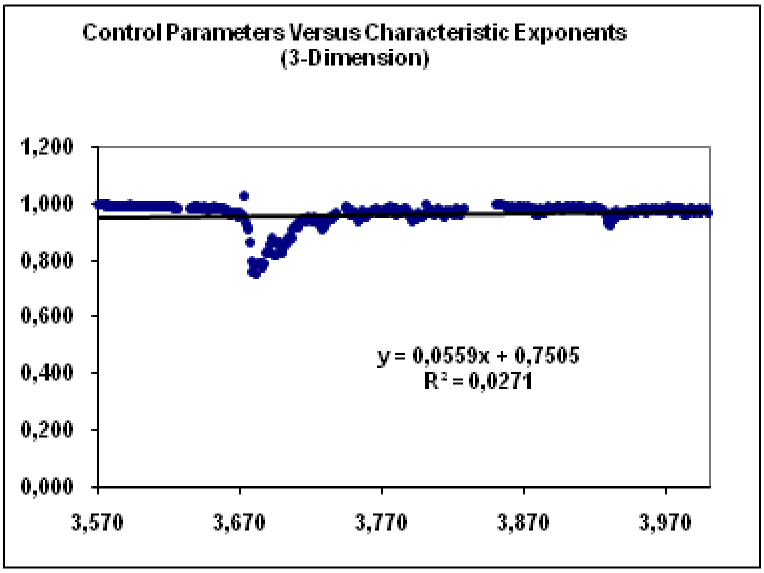

Figure 7. *Control Parameters versus Characteristic Exponents (3-Dimension)*

The step size can be fixed value (1.0) or picked randomly between 0 and 1.0 as explained above. The corresponding β values obtained from invoking equation (8) on particle *A*' for all *K*-values that lead to chaotic solutions are contained in Fig. 5, 6 and 7. Fig. 5 is the results obtained for particle *A*' in 1-D Euclidean space as explained above. Fig. 6 is the results obtained for particle *A*' in 2-D Euclidean space as explained above. Fig. 7 is the results obtained for particle *A*' in 3-D Euclidean space as explained above. We have explained the practical value of β for low and high values. Higher β signify fast diffusion in the studied space and vice versa for lower β.

4. DISCUSSIONS

4.1 General

Remember that an important objective of the work is for the model developed to be able to track chaos in the movement of sawdust particles or show an otherwise behavior. The basic idea is that the model is tested for particle movement in 1-D, 2-D and 3-D directions. One dimension (1-D) visualizes the sawdust particle to be moving in a plane in which a forward movement is allowed. This is not a chaotic situation since no backward and side way movements are allowed. Two dimensional (2-D) movements involve front and side motion of the sawdust particles. Three dimensional (3-D) movements of the sawdust particles involve front movement, a reverse movement (back) and a side movement of the sawdust particles. This is largely chaotic in nature.

If the movement of the sawdust particle is chaotic, this may have been influenced by the wind effect on the sawdust particle and its direction, gravitational force on the sawdust particles and the buoyancy effect on sawdust particles. In simulating the behavior of sawdust particles, fig. 3 was obtained using random number generator algorithm, (ran(seed)). The seed value used was 9876 and the first 100 values returned were taken to be the run-off. It can be observed that the solution almost filled up the plane defined by 1 unit. This has been represented by 500-consecutive steps of a random walk in 2-dimensions. Fig. 4 shows a chaotic behavior of the sawdust particles in 500-consecutive steps.

Referring to Table 1, only 389 out of 3999 sampled control parameter points lead to chaotic solution. Thus, about 9.73% of the control parameter range exhibit chaoticness. Furthermore and referring to Table 1, the listed Logistic equation controls parameters in the range of 3.570 to 3.999 leads to chaotic solution. This is evident from the respective positive Lyapunov exponents. Furthermore the range of the characteristic exponents increases with increasing space dimension. The highest 0.754 to 1.026 was recorded for 3-dimensional Euclidean space. None of the diffusive characteristic exponents obtained is lower than or equal to 0.5 supported by literature for diffusive characteristic exponent of a Random Walk in any Euclidean space. Referring to Fig. 5, there is very little correlation between the variation of the control parameters and the corresponding chaotic walk characterization exponents. The correlation coefficient is low (R^2=0.3525). Referring to Fig. 6 there is very little correlation between the variation of the control parameters and the corresponding chaotic walk characterization exponents. The correlation coefficient is low (R^2=0.0664).

Furthermore the characterization exponents are almost the same for all the parameters except for some parameters in the neighborhood of parameter 3.679. Referring to Fig. 7, there is very little correlation between the variation of the control parameters and the corresponding chaotic Walk characterization exponents. The correlation coefficient is low (R^2=0.0271). Furthermore the characterization exponents are almost the same for all the parameters except for some parameters in the neighborhood of parameter 3.685. For the problem encountered with constant step size, we state that the attempt made to reproduce table 1 and figures 3 to 5 for chaotic walker with constant step size failed, but specifically for some chaotic parameter points and only in 1-dimensional Euclidean space. The K-value of 3.571 in equation (1) was noted for this problem all other setting being equal. The explanation found was that the D_{av} in equation (3) for the 200-walks involved in this study sum to zero for all elapsed time steps that are even numbers. But zero argument is not acceptable for logarithm as required for the estimation of β in equation (3).

4.2 Model verification results

The model verification exercise, which entails, running the same program as done for the original equations (2), (7) and (8) when equation (4) is substituted for (2), yielded interesting results. This variant of analysis did not yield chaotic solution (i.e. for ΔK=0.001 and K=0.001 to K=1.537 in step of 0.001); the average Lyapunov has to be positive for a chaotic solution to occur, but all computed average Lyapunov were negative. From K = 1.538 upward, the program used for the analysis encountered “math overflow error” which indicates non-feasible region of solution. Thus, the methodology has been used to differentiate a chaotic solution based equation from a non-chaotic one. Additional verification exercise was conducted with equation (5) substituted for (2), where equations (5), (7) and (8) now represents the framework to be verified. The same result was obtained compared with the model combination of equations (4), (7) and (8). The specific finding is that between K=0.001 and K=0.434 (inclusive), no chaotic solution was formed. Beyond this range of values, “math overflow error” was observed, indicating non-feasible area of operation.

4. CONCLUSIONS

In this study, an approach has been presented that aids the understanding of how sawdust particles move about in the sawmill environment as a result of the cutting and sawing actions of the machine on wood. The study presented mathematical formulations with linkages from logistic equation to Lyapunov exponent and dispersion characterizing exponents. The model is then validated with variants of logistic models incorporated into the framework to demonstrate its ability to detect chaos or otherwise in any tested model. Existing literature values are also compared to the results obtained. This study showed that about 9.73% of the control parameter range sampled exhibit chaoticness. The study also showed that the dynamics of chaotic walker in any Euclidean space dimension is super-diffusive. The minimum diffusive indicator recorded was 0.754. The study further showed that super-diffusiveness evident as possible supplement confirmation of chaotic behavior of a dynamical system. In addition, table 1 can serve reference purposes, as literature reports on chaotic behavior of Logistic equation are often presented in graphic.

There seems to be a wide variety of investigations that the success of the current paper could be extended to. An immediate application may be to other hazardous jobs such as cement bags loading and off-loading from vehicle in an enclosed warehouse where very little air circulation is permitted. Other related jobs where particles are emitted in activities could also benefit from the application of the study. In addition, it may find suitable application in iron and steel making where alloying of elements may be studied with respect to movement of elements during alloying process. Since the originality of the work lies in the application of a uniquely combined model framework with components largely established in the computational field to a completely new setting with very limited applications, there is obviously a large set of models that could be adapted from literature to current work and then compared with the results presented here. From the literature cited, the diverse approaches could be incorporated into the existing framework and correlation analysis carried out. Thus, the new set of studies may emerge that would add value to literature in order to further our understanding of important aspects concerning sawdust particle movements in sawmill environment.

Nomenclature

n	time
X_n	normalized population of sawmill workers that inhale sawdust particles
X_{n+1}	normalized population of sawmill workers that inhale sawdust particles at time (n+1).
K	level of sawdust spread control put in place
$f(X_n)$	function of X_n
D_{av}	average distance traveled away from the machine that cuts logs into planks
T	elapsed time
β	dispersive characteristic exponent
N	number of cases considered
X_0	initial normalized population of sawmill workers that inhale sawdust particles at time 0.

Definitions of variables used in the flowchart

Step	step at which parameter axis was investigated, value used for this study is 0.001
Nbg	beginning counter for investigation along parameter axis, value used for this study is one (1)
Nend	end counter for investigation along parameter axis, value used for this study is four thousand (4000)
Itr	end counter for computation of transition solution (100)
Nstep	total number of steps investigated for chaotic walks (2000)
Nwalkers	total number of time that 2000-Nstep distinct steps was performed (200)
Iseed	seed value for random step size generation (9876 used)
RR	initial or starting solution of logistic model (0.3 used)
Rate	logistic model parameter (the same as product of float(kkk) and step). kkk takes value from Nbg to Nend at an increment of one (1)
XX	Rate*xx*(1.0-xx)
SumLyExp	Lyapunov Exponent
Xmin	minimum steady solution possible from the logistic model
Xmax	maximum steady solution possible from the logistic model
Expt	Abs(Rate*(1.0-2.0*xx)), Abs=absolute
Xrange	Difference between Xmax and Xmin
X	x-coordinate of chaotic walks
Y	y-coordinate of chaotic walks
Z	z-coordinate of chaotic walks
Ratte	Subroutine for computing the diffusive characteristic exponent and intercept
Xm	mean value of logarithm of scales (i.e. mean of 1 to 2000)
Ym	mean value of logarithm of average distance of 200 attempts at the end of each step in 2000-chaotic step (step size varies randomly)
Dmean(i)	average distance value for i=1 to 2000 communicated to the subroutine by the main programme
Dm	characteristic exponent
Cm	Intercept

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