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MODELING OF ENERGY CONSUMPTION FOR ONE SIDED HEATING OF WOOD DETAILS BEFORE THEIR BENDING, IN PRODUCTION OF STRINGED MUSIC INSTRUMENTS

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ABSTRACT

A mathematical model and numerical approach to computation of the specific energy consumption, which is needed for one sided heating of flat wood details aimed at their plasticizing in the production of curved outside parts for corpuses of stringed music instruments, have been suggested. The approach is based on integration of the solutions of a linear model for calculation of the non-stationary 1D temperature distribution along the thickness of flat wood details subjected to one sided heating.

For numerical solution of the model a software program has been prepared, which has been input in the calculation environment of Visual Fortran Professional. Using the program, computations have been carried out for determination of the change in specific energy, which is consumed by spruce details with an initial temperature of 20 ^{o}C, moisture content of 0.15 kg·kg^{-1}, and thicknesses of 6 mm, 8 mm, and 10 mm during their 10 min one sided heating at temperatures of the heating body of 100 ^{o}C, 120 ^{o}C, and 140 ^{o}C and of the surrounding air of 20 ^{o}C. The obtained results have been graphically presented and analyzed.

Key words: spruce details, one sided heating, plasticizing, bending, specific energy consumption, Visual Fortran

1. INTRODUCTION

An important component of the technologies for production of curved wood details is their plasticizing up to the stage that allows their faultless bending.

Energy consumption for one sided heating of details aimed at their plasticizing before bending depends on many factors: wood specie, thickness and moisture content of the details, temperatures of the heating medium and of the surrounding air, desired degree of plasticizing and radius of the bending, etc. (Chudinov, 1968); (Shubin, 1990); (Taylor, 2001); (Trebula and Klement, 2002); (Pervan, 2009); (Angelski, 2010); (Deliiski and Dzurenda, 2010); (Gaff and Prokein, 2011).

One sided heating is applied, for example, in production of curved outside parts for corpuses of string music instruments so that they are plasticized before bending. In practice those details are with thicknesses between 5 mm to 10 mm and with moisture content around 15%.

The technology for plasticizing such details has been using press equipment with metal band, electrically heated up to temperature in range between 100 ºC ÷ 150 ºC (Fig. 1).

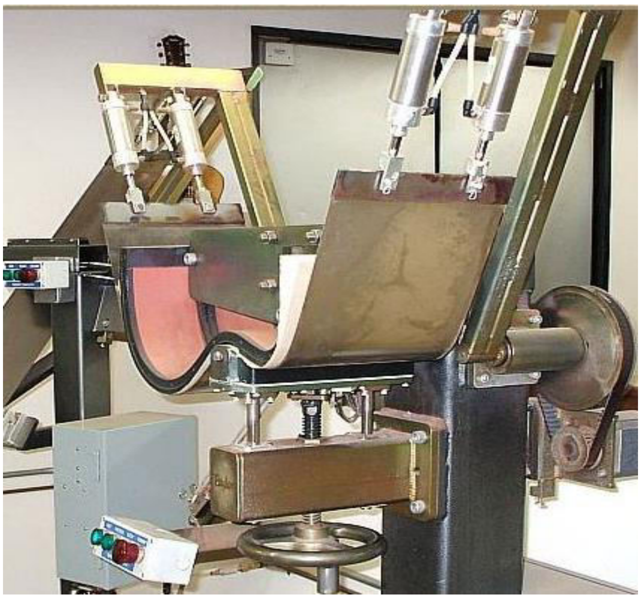


Figure 1. *Equipment with electrically heated band for one sided heating and bending of flat wood details in production of outside parts for corpuses of stringed music instruments*

In specialized literature there is no information at all about temperature distribution in wood details during their one sided heating and about energy consumption needed for realization of such heating.

The aim of the present work is to suggest a mathematical model and numerical approach to computation of the specific energy consumption, which is needed for one sided heating of flat wood details aimed at their plasticizing in the production of curved outside parts for corpuses of stringed music instruments. The approach has to be based on integration of the solutions of a linear model for calculation of non-stationary 1D temperature distribution along the thickness of flat wood details subjected to one sided heating.

2. MATERIAL AND METHODS

Modeling of heat distribution in flat wood details during their one sided heating

When the width of wood details exceeds their thickness by at least 3 ÷ 4 times, then calculation of change in temperature only along thickness of the details during their one sided heating (i.e. along the coordinate x, which coincides with the thickness h) can be carried out by means of the following linear 1D mathematical model (Deliiski, 2003):

$$\frac{\partial T(x,\tau)}{\partial \tau} = a_{\mathrm{c}} \frac{\partial^2 T(x,\tau)}{\partial x^2} \tag{1}$$

with an initial condition

$$T(x,0) = T_0 \tag{2}$$

and the following boundary conditions:

• from the side of the details' heating – at prescribed surface temperature, which is equal to the temperature of the metal heating body T_{m}:

$$T(0,\tau) = T_{\mathrm{m}}(\tau)\,; \tag{3}$$

• from the opposite non-heated side of the details – at convective heat exchange between the details' surface and the surrounding air environment

$$\frac{\partial T(X,\tau)}{\partial x} = -\frac{\alpha(\tau)}{\lambda_{\mathrm{s}}(\tau)}\left[T_{\mathrm{a}}(\tau) - T_{\mathrm{s}}(\tau)\right] . \tag{4}$$

For practical usage of eqs. (1) and (4) it is needed to have mathematical descriptions of the wood temperature conductivity cross sectional to the fibers, a_c, of the wood thermal conductivity cross sectional to the fibers, $\lambda_c = \lambda_s$ and of the heat transfer coefficient between the details' surface at their non-heated side and the surrounding air, α. For this purpose, the description of a_c and λ_c given in (Deliiski, 2003) can be used.

Calculation of the heat transfer coefficient α can be carried out with the help of the following equation, which has been suggested by Chudinov (1966, 1968), for cases of cooling horizontally situated wood plates in atmospheric conditions of free convection:

$$\alpha = 3.256\left[T_{\mathrm{s}}(\tau) - T_{\mathrm{a}}(\tau)\right]^{0.25} . \tag{5}$$

According to eq. (3), the temperature at the detail's surface being in contact with the heating body (i.e. the characteristic point with *x-coordinate* = 0 mm) is equal to its temperature T_m, due to the extremely high coefficient of heat transfer between the body and the wood during their very close contact.

Modeling of specific energy consumption for one sided heating of wood details

It is known that specific energy consumption for heating of 1 m^3 solid materials with an initial mass temperature T_0 to a given mass temperature T_{av} is determined using the equation (Deliiski, 2003, 2013b)

$$q = \frac{c \cdot \rho \cdot \left(T_{\mathrm{av}} - T_0\right)}{3.6 \cdot 10^6} , \tag{6}$$

where

$$T_{\mathrm{av}} = \frac{1}{h}\int_{(h)} T(x,\tau)\mathrm{d}h , \tag{7}$$

The multiplier $3.6 \cdot 10^6$ in the denominator of eq. (6) ensures that the values of q are obtained in kWh·m^{-3}, instead of in J·m^{-3}.

For practical usage of eq. (6), it is needed to have mathematical descriptions of the specific heat capacity of non-frozen wood, c, and of wood density, ρ. Such descriptions are given in (Deliiski, 2003); (Deliiski and Dzurenda, 2010) and in (Deliiski, 2013a), respectively.

Transformation of mathematical models in suitable form for programming

Presentation of the mathematical model (1) ÷ (5) through its discrete analogue suitable for programming corresponds to the one shown in Fig. 2, setting of the coordinate system and positioning of the nodes in the calculation mesh, in which the 1D distribution of temperature along the thickness of flat wood details subjected to one-sided heating is computed.

The following system of equations has been derived by passing to final increases of the derivatives in equations (1) ÷ (5) with application of the same explicit form of the finite-difference method, which has been described in (Deliiski, 2003), (Deliiski 2011), (Deliiski 2013b); (Deliiski and Dzurenda, 2010).

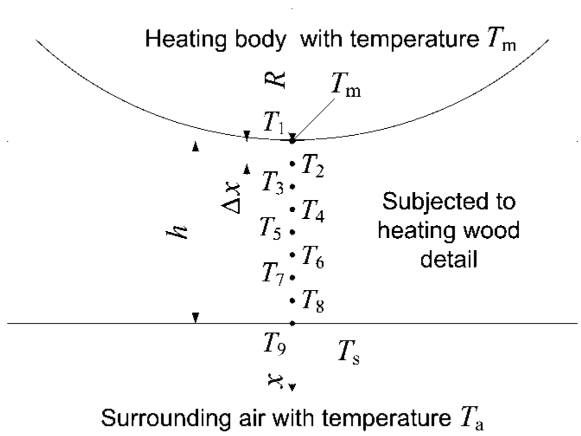


Figure 2. *Positioning of the nodes of 1D calculation mesh in a discretized detail's thickness*

$$T_i^{n+1} = T_i^n + \frac{a_c \Delta\tau}{\Delta x^2}\left(T_{i-1}^n + T_{i+1}^n - 2T_i^n\right), \tag{8}$$

$$T_i^0 = T_0 \quad @ \quad 1 \le i \le \mathrm{M} = 9, \tag{9}$$

$$T_1^{n+1} = T_m^{n+1}, \tag{10}$$

$$T_9^{n+1} = \frac{T_8^n + \dfrac{\alpha^n \cdot \Delta x \cdot T_a^n}{\lambda_s}}{1 + \dfrac{\alpha^n \cdot \Delta x}{\lambda_s}}, \tag{11}$$

$$\alpha^n = 3.256\left(T_9^n - T_a^n\right)^{0.25}, \tag{12}$$

where

$$\Delta x = \frac{h}{M-1}, \tag{13}$$

$$M = \frac{h}{\Delta x} + 1, \tag{14}$$

$$\Delta\tau \le \frac{\Delta x^2}{a_c}. \tag{15}$$

The mathematical model of the specific energy consumption for one sided heating of wood details, which is presented by eqs. (6) and (7), obtains the following form suitable for programming:

$$q^{n+1} = \frac{\rho \cdot c}{3.6 \cdot 10^6}\left(T_{av}^{n+1} - T_0\right), \tag{16}$$

where

$$T_{av}^{n+1} = \int_{(h)} T[x,(n+1)\Delta\tau]\mathrm{d}h. \tag{17}$$

Highest precision for solution of the eq. (17) is ensured by Simpson's method (Deliiski, 2003). According to this method and to Figure 2, eq. (17) obtains the following discrete analogue (Dorn, McCracken, 1972):

$$T_{\text{av}}^{n+1} = \frac{\Delta x}{3}\left(T_1^{n+1} + 4T_2^{n+1} + 2T_3^{n+1} + 4T_4^{n+1} + 2T_5^{n+1} + 4T_6^{n+1} + 2T_7^{n+1} + 4T_8^{n+1} + T_9^{n+1}\right). \quad (18)$$

3. RESULTS AND DISCUSSION

For numerical solution of the above presented mathematical models, a software program was prepared in FORTRAN, which was input in the calculation environment of Visual Fortran Professional developed by Microsoft.

With the help of the program, as examples, computations were made to determine the specific energy consumption, which is needed for one sided heating of non-frozen spruce (*Picea Abies Karst*) details with thicknesses of h = 6 mm, h = 8 mm, h = 10 mm, initial wood temperature of t_0 = 20 °C, and wood moisture content of u = 0.15 kg·kg^{-1} during their 10 min heating at t_m = 100 °C, t_m = 120 °C, t_m = 140 °C, and t_a = 20 °C.

The computations were done with average values of spruce temperature conductivity cross-sectional to the fibers, a_c, and of spruce thermal conductivity cross-sectional to the fibers, $\lambda_c = \lambda_s$, which were obtained using the mathematical descriptions of a_c and λ_c depending on temperature, wood moisture content and fiber saturation point of the wood species (Deliiski, 2003).

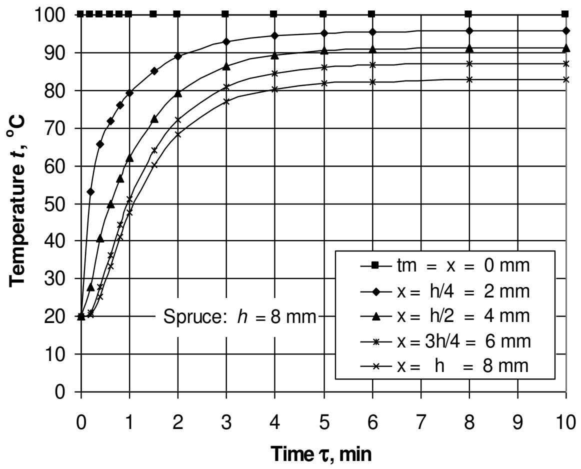
The calculated values of a_c and λ_c with the help of these mathematical descriptions for spruce wood with u = 0.15 kg.kg^{-1} and fiber saturation point $u_{\text{fsp}}^{293.15} = 0.32\ \text{kg}\cdot\text{kg}^{-1}$ (Videlov, 2003); (Deliiski and Dzurenda, 2010) in temperature ranges from 20 °C to 100 °C, from 20 °C to 120 °C, and from 20 °C to 140 °C, are shown in Table 1.

The linear dependences of a_c and of $\lambda_s = \lambda_c$ on t (Deliiski, 2003) allow for solution of the mathematical model using the average arithmetic values of a_c and $\lambda_s = \lambda_c$ in respective temperature ranges (Table 1) for determination of temperature distribution along the details' thickness during one sided heating of the details.

Table 1. *Change in a_c, $\lambda_c = \lambda_s$, and c of spruce wood with u = 0.15 kg.kg^{-1}, depending on t*

Parameter of the wood	Temperature t, °C				Average arithmetic values of λ_c and a_c for temperature ranges:		
	20	60	100	140	t = 20 ÷ 100 °C	t = 20 ÷ 120 °C	t = 20 ÷ 140 °C
$\lambda_s = \lambda_c$, W·m^{-1}·K^{-1}	0.2341	0.2664	0.2987	0.3311	0.2664	0.2745	0.2826
$a_c \cdot 10^7$, m^2·s^{-1}	2.5799	2.7412	2.8818	3.0052	2.7331	2.7627	2.7926
c, J·kg^{-1}·K^{-1}	2036	2181	2326	2472	2181	2218	2254

Figure 3 presents the temperatures of the heating body t_m = 100 °C and t_m = 140 °C, which have been entered based on the input data used for solution of the 1D model. This figure also shows temperature change calculated by the model in 4 equidistant from one other characteristic point along the thickness of the details with thickness of h = 8 mm during their one sided heating. The coordinates of those points are shown in the legends of the figure.



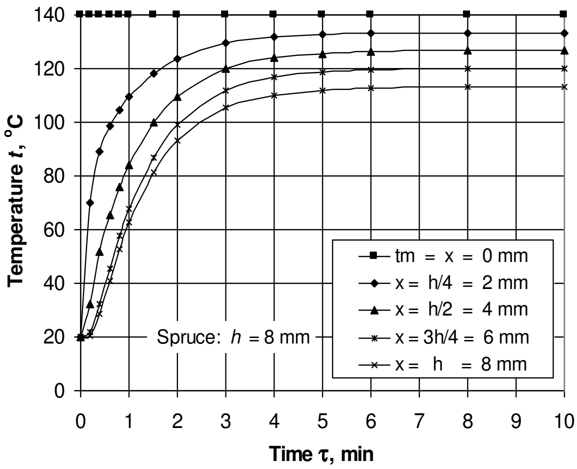


Figure 3. *Change in t along thickness of spruce details with $t_0 = 20\ ^oC$, $u = 0.15\ kg.kg^{-1}$, and $h = 8$ mm during their one sided heating at $t_m = 100\ ^oC$ (left) and $t_m = 140\ ^oC$ (right) at $t_a = 20\ ^oC$*

Simultaneously with the solution of the 1D model, calculations of t_{av} and q were carried out, using the value of density ρ = 445.6 kg.m^{-3} and the average arithmetic values of the specific heat capacity c given in the last row of Table 1, in respective temperature ranges. This value of ρ is calculated according to the mathematical description of wood density in hygroscopic range given in (Deliiski, 2013a) for spruce wood with $u = 0.15$ kg·kg^{-1}, $u_{fsp} = 0.32$ kg·kg^{-1}, basic density $\rho_b = 380$ kg·m^{-3}, and volume shrinkage $S_v = 11.4\%$ (Videlov, 2003). The values of c are calculated according to the mathematical description of the specific heat capacity of wood in hygroscopic range given in (Deliiski, 2003); (Deliiski, 2011); (Deliiski , 2013b). Because of the linear dependence of c on t, the average arithmetic values of c for the respective temperature ranges were used for finding the solution of eq. (16).

Fig. 4, 5 and 6 represent the calculated change of q during one sided heating of spruce details with studied thicknesses at $t_m = 100$ °C, $t_m = 120$ °C, and $t_m = 140$ °C respectively.

The results obtained show that through one sided heating of the details, the change of t, t_{av}, and q go on following complex curves. By increasing the heating time, the curves of t gradually approach asymptotically their highest values, decreasingly dependent on the remoteness of the characteristic points from the heated surface of the details (refer to Fig. 3). Analogously, the curves of the change in t_{av} and q approach asymptotically to their highest values, increasingly dependent on t_m and decreasingly dependent on h.

The highest values of t, t_{av}, and q are achieved when a stationary temperature distribution occurs along the details' thickness.

The specific energy consumption q reaches its highest values when at given values of h, t_m, and t_a a stationary temperature distribution along the thickness of the wood details subjected to one sided heating occurs, as follows:

• for $h = 6$ mm: 19.65 kWh·m^{-3} after 3.5 min heating at t_m=100 °C, 24.98 kWh·m^{-3} after 3.8 min heating at t_m= 120 °C, and 30.44 kWh·m^{-3} after 3.9 min heating at t_m= 140 °C;

• for $h = 8$ mm: 19.27 kWh·m^{-3} after 6.4 min heating at t_m=100 °C, 24.37 kWh·m^{-3} after 6.7 min heating at t_m= 120 °C, and 29.71 kWh·m^{-3} after 6.8 min heating at t_m= 140 °C;

• for $h = 10$ mm: 18·81 kWh.m^{-3} after 9.7 min heating at t_m=100 °C, 23.85 kWh·m^{-3} after 10.2 min heating at t_m= 120 °C, and 29.07 kWh·m^{-3} after 10.4 min heating at t_m= 140 °C.

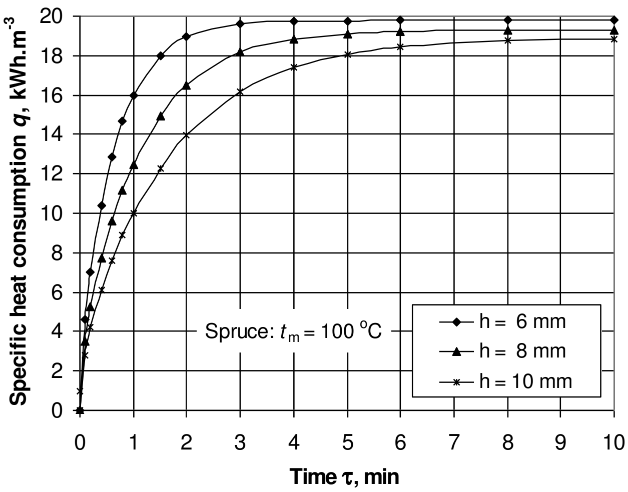


Figure. 4. *Change in q for spruce details with t_0 = 20 °C, u = 0.15 kg.kg^{-1}, during their one sided heating at t_m = 100 °C and t_a = 20 °C, depending on h*

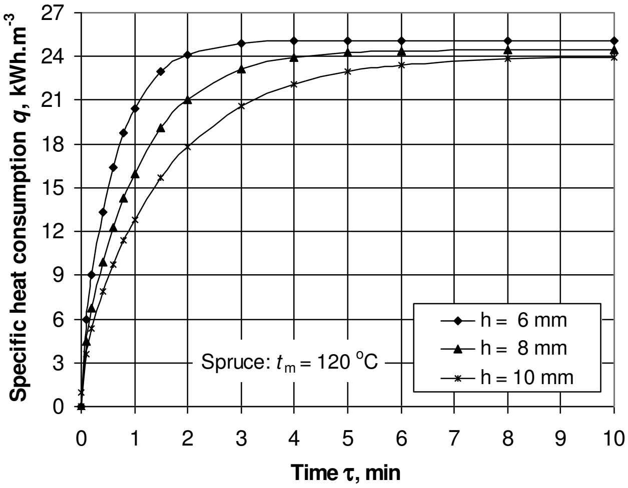


Figure. 5. *Change in q for spruce details with t_0 = 20 °C, u = 0.15 kg.kg^{-1}, during their one sided heating at t_m = 120 °C and t_a = 20 °C, depending on h*

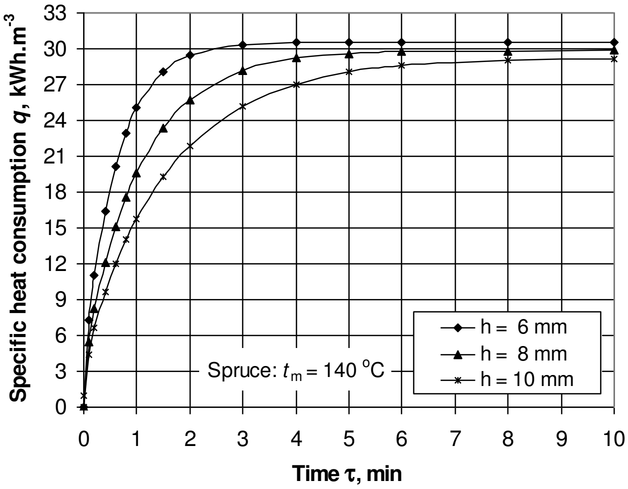


Figure. 6. *Change in q for spruce details with t_0 = 20 °C, u = 0.15 kg.kg^{-1}, during their one sided heating at t_m = 140 °C and t_a = 20 °C, depending on h*

4. CONCLUSIONS

The paper describes a mathematical model and a numerical approach for computation of the specific energy consumption needed for one sided heating of flat wood details aimed at their plasticizing in the production of curved outside parts for corpuses of stringed music instruments. The approach is based on integration of the solutions of a linear mathematical model for calculation of the non-stationary 1D temperature distribution along the thickness of flat wood details subjected to one sided heating

For the numerical solution of the models a software program was prepared, which was input in the calculation environment of Visual Fortran Professional. As examples for use of the models and the suggested approach, computations were carried in order to determine the change in the specific energy which is consumed by spruce details with an initial temperature of 20 ^{o}C, moisture content of 0.15 kg.kg^{-1}, and thicknesses of 6 mm, 8 mm, and 10 mm during their 10 min one sided heating at temperatures of the heating body 100 ^{o}C, 120 ^{o}C, and 140 ^{o}C and of the surrounding air 20 ^{o}C.

The obtained results show that during the one sided heating of the details, the change in the specific energy, which is consumed by the details, q, goes on according to complex curves. The results also show that by increasing the heating time the curves of q gradually approach asymptotically the highest values of q, which increasingly depend on t_m and decreasingly depend on h.

Specific energy consumption q reaches its highest values when at given values of h, t_m, and t_a a stationary temperature distribution along the thickness of the wood details subjected to one sided heating occurs. For example, when spruce details with t_0 = 20 ^{o}C and u = 0.15 kg.kg^{-1} are heated at t_m = 100 ^{o}C and t_a = 20 ^{o}C, the highest values of q = 19·65 kWh.m^{-3} at h = 6 mm, of q = 19·27 kWh.m^{-3} at h = 8 mm, and of q = 18·81 kWh·m^{-3} at h = 10 mm occur after the beginning of the heating of 3.5 min, 6.4 min, and 9.7 min, respectively. When the same details are heated at t_m = 120 ^{o}C and t_a = 20 ^{o}C, the highest values of q = 24.98 kWh·m^{-3} at h = 6 mm, of q = 24.37 kWh·m^{-3} at h = 8 mm, and of q = 23.85 kWh·m^{-3} at h = 10 mm occur after the beginning of the heating of 3.8 min, 6.7 min, and 10.2 min, respectively.

The obtained results can be used for a science-based determination of energy consumption, which is needed for one sided heating of flat wood details aimed at their plasticizing before bending in the production of curved details for different applications in furniture and other industries. They are also of specific significance for optimization of the technology and of the model-based automatic control (Deliiski, 2003); (Hadjiiski, 2003); (Deliiski and Dzurenda, 2010) of the heating process of details before their bending.

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Symbols

a = temperature conductivity (m^2·s^{-1})
c = specific heat capacity (J·kg^{-1}·K^{-1})
h = thickness (m)
I = number of nodal points in the direction along the detail's thickness: i = 1, 2, 3,…, M
M = total number of nodal points along the detail's thickness: M = $h/\Delta x$+1
n = time level during the solution of the mathematical model: n = 0, 1, 2, 3, …..
q = specific energy consumption (kWh·m^{-3})
R = radius of bending of heated and plasticized wood detail (m)
S = wood shrinkage (%)
t = temperature (^{o}C): $t = T - 273.15$
T = temperature (K): $T = t + 273.15$
u = moisture content (kg·kg^{-1}): $u = W/100$

W = moisture content (%): $W = 100u$
x = coordinate along the thickness of the details: $0 \leq x \leq X = h$
α = heat transfer coefficient ($W \cdot m^{-2} \cdot K^{-1}$)
λ = thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)
ρ = density ($kg \cdot m^{-3}$)
τ = time (s)
Δx = step on the x-coordinate, which coincides with the thickness of the details (m)
$\Delta \tau$ = step on the τ-coordinate, i.e. interval between time levels (s)
@ = at

Subscripts and superscripts:
a = air (for temperature of the air close to the non-heated side of wood details)
av = average (for the average mass temperature of the details at given moment of their one sided heating or for the average arithmetic values of the thermo physical characteristics of wood)
b = basic (for density, based on dry mass divided to green volume)
c = cross-sectional to the fibers (for the values of the thermo physical characteristics of wood)
fsp = fiber saturation point of wood
m = medium (for the temperature of the heating metal body)
max = maximum (for the largest values of t and q)
s = surface (for the non-heated surface of the wood details)
v = volume (for wood shrinkage)
0 = initial (for the average mass temperature of the details at the beginning of the heating or for the time level at the beginning of the model's solution)
293.15 = at 293.15 K, i.e. at 20 ^{o}C (for standard values of wood fiber saturation point)

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