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INVESTIGATION OF THE NATURAL FREQUENCIES AND THE MODE SHAPES OF CIRCULAR SAW WITH COMPENSATING SLOTS BY THE FINITE ELEMENT METHOD

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ABSTRACT

This paper shows the methodics and results of the simulative investigation of the circular saw with compensating slots. The investigation is an extension of the previous one done by the authors. The natural frequencies and mode shapes of this kind of circular saws are obtained as results of the investigations. The estimation is done by application programme Cosmos Works. Physical and mechanical properties of the materials are taken into account. The adequate mechanic-mathematical model is used for the aims of the study. The typical characteristics of the structure of this kind of circular saws were taken into account in the model. The circular saw is drawn in 3D by the application programme Solid Works and it is modeled with four nodes 3D finite elements. The results of this investigation prove the practical significance of the model. They point the possibilities for determinations of resonant regimes and they are basis for their detailed studying.

Keywords: circular saws, modeling, vibrations

1. INTRODUCTION

The current circular saws are made of high quality steel with excellent mechanical characteristics. Therefore, during their constructive formation it is possible to foresee slots which improve the work of the saw. In addition, tungsten carbide teeth (TCT) can be included. They increase the wear resistance at the cutting. They are soldered to the saw's body by special technologies and they provide the necessary reliability during its work, as well as the quality of the product processing. An important condition in the production of the circular saws is reaching high accuracy of its designing shape and measures. This condition requires usage of current (more often laser) technologies for processing of the tool. Usage of current materials and technologies is a premise for making the circular saws which have qualities that are necessary for the practice. These saws allow intensification of the work process, using higher speed during cutting. This process has advantages but there are some problems. One of them is the excessive heating of the saw during its work. It can lead to deformations of the disk and damage the accuracy and quality of the processing of the production. To avoid that, it is necessary to make compensating slots in the saw's body. A circular saw with compensating slots is shown in Figure 1. The slots are formed in order to avoid deformations, thus saving the high quality of the cutting of the saw even in hard conditions (significant centrifugal forces, heating caused by the friction between the saw and the wood).

The existence of the compensating slots in the circular saw's body influences the frequencies of the natural vibrations and mode shapes of the circular saw. This impacts the tasks of the designing and measuring of the whole circular machine (Vukov, Gochev, Slavov 2010), (Vukov, Georgieva 2009), (Obreshkov, 1996). It is necessary to do beforehand some estimations of the danger of the resonance.

Therefore, it's necessary to investigate the natural frequencies and mode shapes of the circular saw. It leads to simulative investigations which help differ the resonant regimes (Amirouche, 2006), (Coutinho, 2010). They are done on the basis of adequate mechanic-mathematical model, taking into account the typical characteristics of the structure and the physical-mechanical characteristics of the materials of this kind of circular saws (Veits, Kochura, Martinenko, 1971), (Minchev, Grigorov, 1998), (Philipov, 1977).

Figure 1. *Circular saw with compensating slots*

The aim of the study is to build an adequate mechanical-mathematical model for investigation of free vibrations of a kind of circular saws with compensating slots, concerning characteristics in its structure. Some simulative investigations can be made on this basis and these investigations can help to define resonance regimes and to formulate some requirements needed to avoid them.

2. MECHANIC-MATHEMATICAL MODEL OF CIRCULAR SAW WITH COMPENSATING SLOTS

The circular saw with compensating slots, drawn in 3D by the application programme Solid Works (www.solidworks.com), is shown in the Figure 2.

Figure 3 shows the mesh of four node 3D finite elements, modeled by the application programme Cosmos Works.

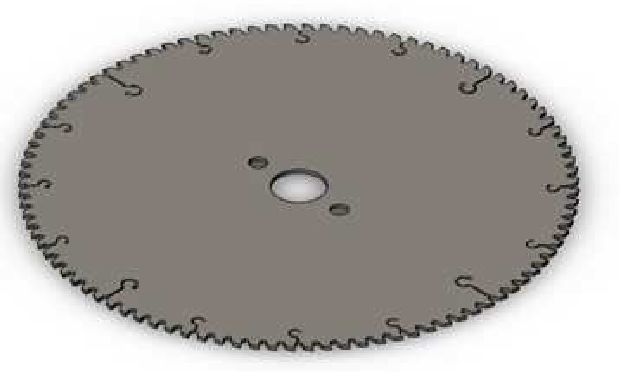


Figure 2. *Circular saw with compensating slots, drawn in 3D*

The investigation of the vibrations of the circular saw with compensating slots requires formulation and solution of the differential equations which describe these processes. Therefore, the matrix mechanics is used (Angelov, 2010), (Angelov, Slavov, 2010).

3. DIFFERENTIAL EQUATIONS

The differential equations which describe the free continuous vibrations of the circular saw are

$$\mathbf{M}.\ddot{\mathbf{q}}+\mathbf{C}.\mathbf{q}=0, \tag{1}$$

Where:

$\mathbf{q}=[q_1 \quad q_2 \quad ... \quad q_n]^T$ is the vector of the generalized coordinates; (2)

$\mathbf{M}$ – the matrix, which characterizes the mass-inertial properties of the mechanical system;

$\mathbf{C}$ – the matrix, which characterizes the elastic properties of the mechanical system.

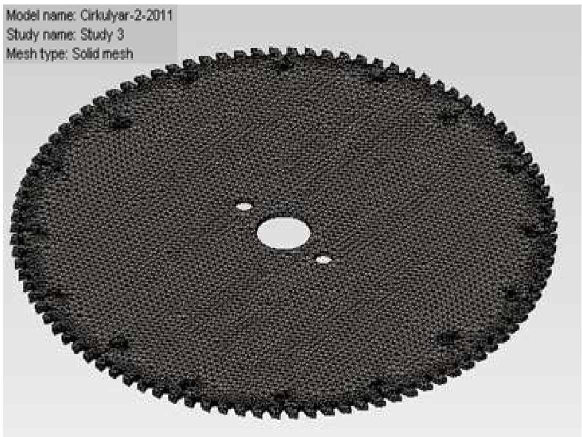


Figure 3. *The circular saw with compensating slots, modeled by the mesh of finite elements*

The system of connected linear differential equations is obtained when the vibrations are small. Particular solutions of the system of the differential equations (1) are searched as:

$$q_r = h_r.\sin(\omega_r.t+\varphi), \tag{3}$$

Where h_r is the amplitude of the small vibration on the generalized coordinate q_r with natural frequency ω_r, and φ is initial phase.

After differentiation of (3) and substituting in (1), a system of linear algebraic equations is obtained:

$$\left|\mathbf{C}-\omega^2.\mathbf{M}\right|.\mathbf{V}=0. \tag{4}$$

To determine the natural frequencies and the mode shapes, it is necessary to solve the task of finding the natural values and the natural vectors of the equations (4). A satisfactory result of the equations (4) requires the following:

$$\det\left(\mathbf{C}-\omega^2.\mathbf{M}\right)=0. \tag{5}$$

The roots of the characteristics equation determine the natural frequencies. The natural frequencies form the matrix of the natural values. It is:

$$\omega = diag\left\lfloor \omega_{r,r} \right\rfloor, \quad r=1,2,...n. \tag{6}$$

The natural frequencies are determined by (6):

$$f_r = \frac{\omega_{r,r}}{2.\pi}\ Hz. \tag{7}$$

The natural values of the system (5) determine the natural vectors. The modal matrix of the free vibrations is determined by the equations (4) and (5):

$$\mathbf{V} = \begin{bmatrix} V_{11} & V_{12} & \dots & V_{1n} \\ V_{21} & V_{22} & \dots & V_{2n} \\ \dots & \dots & \dots & \dots \\ V_{m1} & V_{m2} & \dots & V_{mn} \end{bmatrix} \qquad i = 1...m; \quad j = 1..n, \tag{8}$$

where V_{ij} are the unknown amplitudes of the nodes' moving by free vibrations. The natural frequencies and the mode shapes are determined by the known matrix, which characterizes the mass-inertial properties and the matrix that characterizes the elastic properties of the mechanical system.

4. NUMERICAL INVESTIGATIONS

Numerical investigation is done by modeling of a kind of circular saws with compensating slots by the finite elements method. Physical-mechanical characteristics of materials are taken into account – they are shown in Tables 1, 2 and 3. The estimation of the natural frequencies and mode shapes of the circular saw is done by the application programme Cosmos Works.

Table 1. *Model Information*

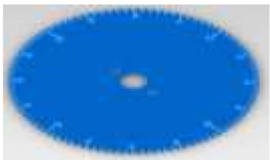
Document Name and Reference	**Treated As**	**Volumetric Properties**
Extrude1 	**Solid Body**	**Mass:1,09228 kg** **Volume:0,000138966 m^3** **Density:7860 kg/m^3** **Weight:10,7043 N**

Table 2. *Mesh Information*

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Jacobian points	4 Points
Mesh Quality	High

Table 3. *Mesh Information - Details*

Total Nodes	82992
Total Elements	40808
Maximum Aspect Ratio	24,242
% of elements with Aspect Ratio < 3	90,0
% of elements with Aspect Ratio > 10	0,0662
% of distorted elements(Jacobian)	0
Time to complete mesh(hh;mm;ss):	00:04:00

5. RESULTS

The first 30 natural frequencies and mode shapes of the studied circular saw are determined. The estimated natural frequencies are shown in Table 4. The results, which illustrate only some of the natural modes, are included because of the limited size of the article. They are shown in Figure 4 and their characteristics and shown in Table 5.

Table 4. *Mode List*

Frequency Number	Rad/sec	Hertz	Seconds
1	0	0	1e+032
2	0	0	1e+032
3	0	0	1e+032
4	0	0	1e+032
5	0	0	921,85
6	0	0	482,23
7	830,67	132,2	0,007564
8	833,09	132,59	0,007542
9	1402,4	223,19	0,0044804
10	1869,9	297,61	0,0033601
11	1871,3	297,83	0,0033576
12	3134,7	498,9	0,0020044
13	3159,6	502,86	0,0019886
14	3284,8	522,79	0,0019128
15	3289,4	523,52	0,0019101
16	4604,1	732,77	0,0013647
17	4638,9	738,3	0,0013545
18	5481,9	872,48	0,0011462
19	5586,6	889,13	0,0011247
20	6193	985,64	0,0010146
21	6233,5	992,09	0,001008
22	6400,3	1018,6	0,00098171
23	7806	1242,4	0,00080492
24	8011,7	1275,1	0,00078425
25	8389	1335,1	0,00074898
26	8415,9	1339,4	0,00074659
27	9208,2	1465,5	0,00068235
28	9512,9	1514	0,00066049
29	9524,4	1515,9	0,00065969
30	10441	1661,7	0,00060181

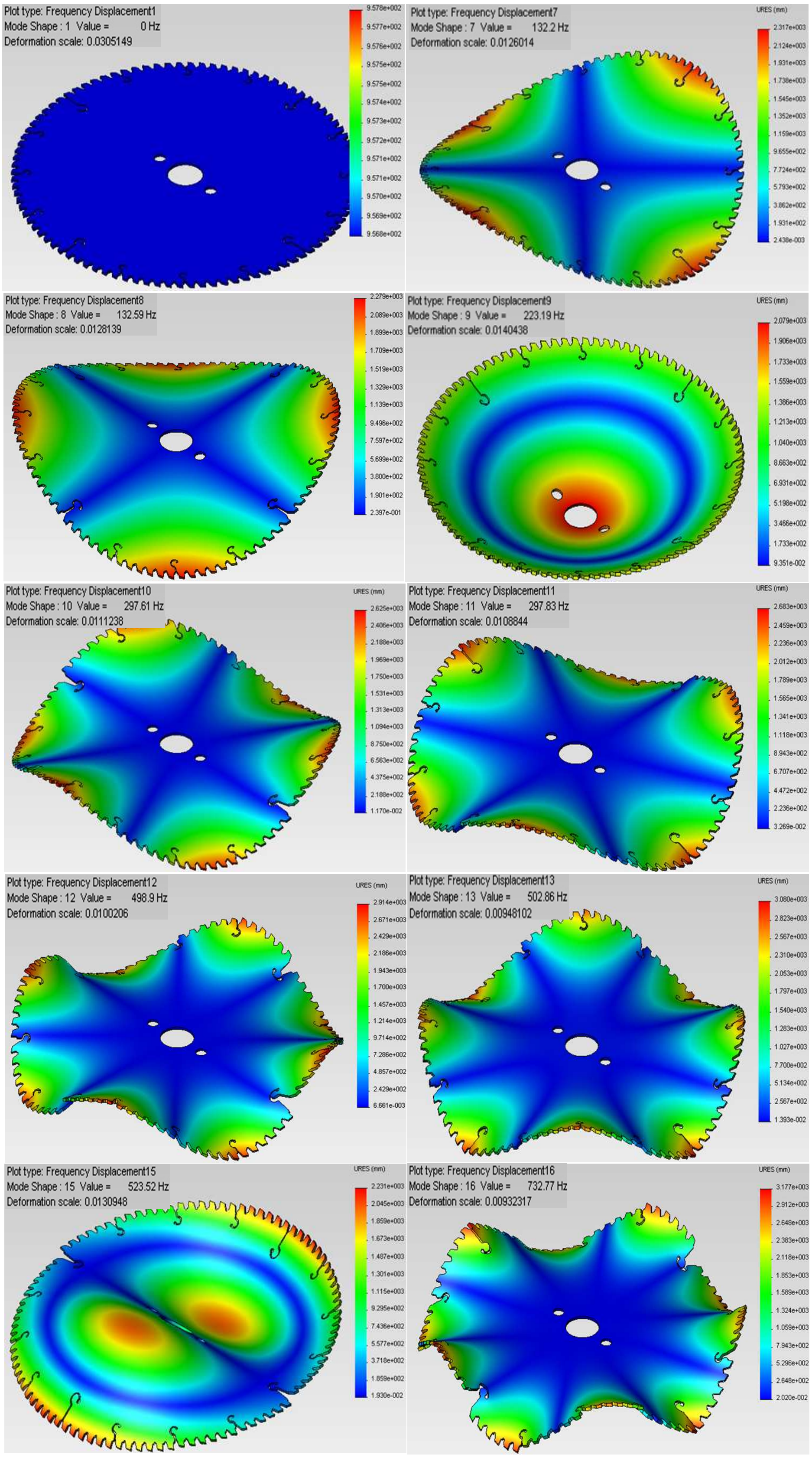


Figure 4. *Mode Shapes*

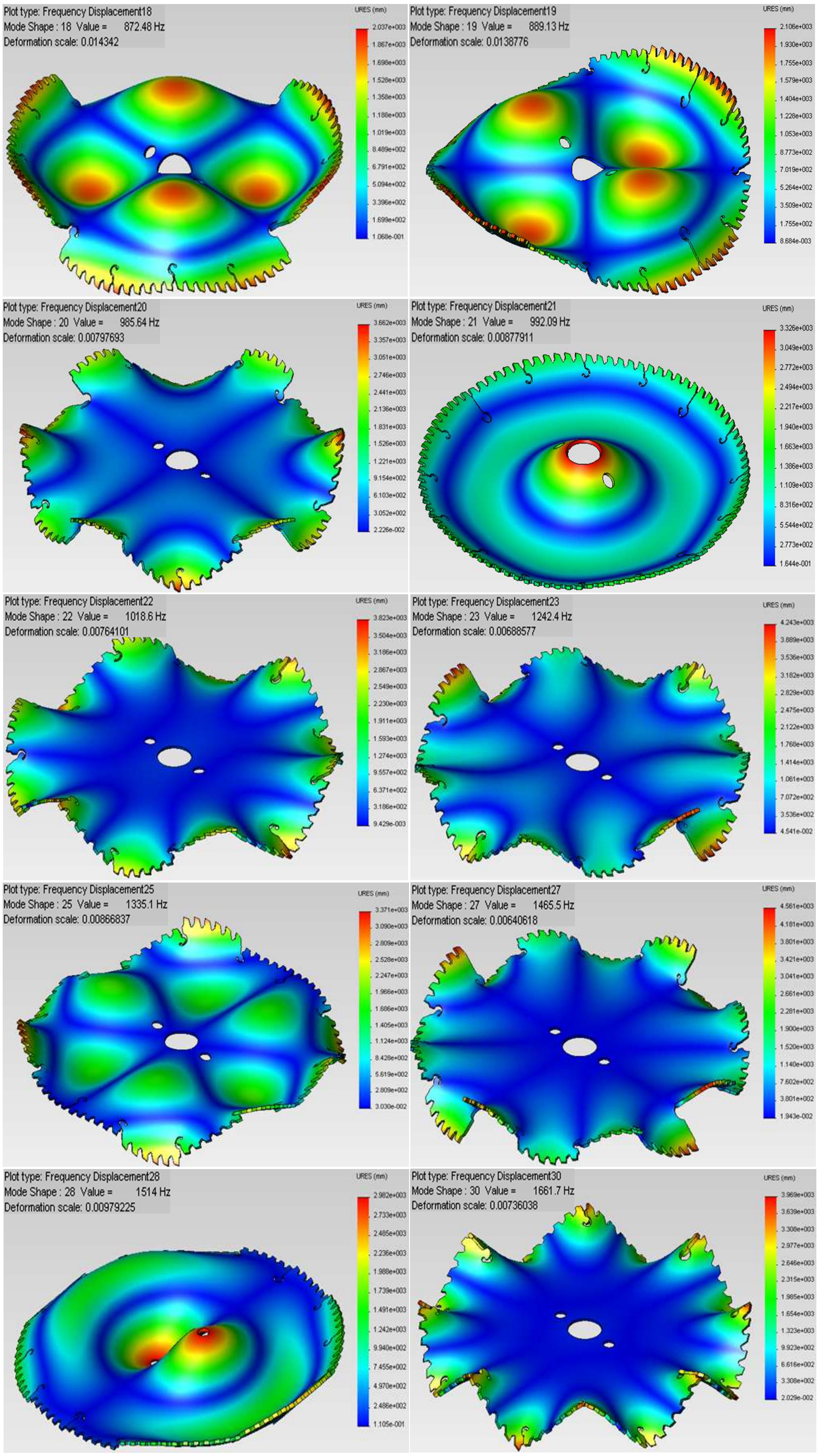


Figure 5. *Mode Shapes*

Table 5. *Characteristics of the natural modes*

Name	Type	Min	Max
Displacement 1	URES: Resultant Displacement Plot for Mode Shape: 1(Value = 0 Hz)	956,821 mm, Node:1	956,821 mm, Node: 1
Displacement 7	Mode Shape: 7 (Value = 132,205 Hz)	0,00243795 mm, Node: 56584	2317,14 mm, Node: 1894
Displacement 8	Mode Shape: 8 (Value = 132,591 Hz)	0,239693 mm, Node: 39302	2278,73 mm, Node: 2163
Displacement 9	Mode Shape: 9 (Value = 223,194 Hz)	0,0935076 mm, Node: 49421	2079,02 mm, Node: 82772
Displacement 10	Mode Shape: 10 (Value = 297,61 Hz)	0,0116976 mm, Node: 42618	2625,07 mm, Node: 1894
Displacement 11	Mode Shape: 11 (Value = 297,829 Hz)	0,0326917 mm, Node: 43541	2682,8 mm, Node: 1345
Displacement 12	Mode Shape: 12 (Value = 498,898 Hz)	0,00666125 mm, Node: 57539	2914,29 mm, Node: 1192
Displacement 13	Mode Shape: 13 (Value = 502,865 Hz)	0,0139329 mm, Node: 39758	3080,15 mm, Node: 1345
Displacement 15	Mode Shape: 15 (Value = 523,521 Hz)	0,0193011 mm, Node: 46026	2230,85 mm, Node: 1818
Displacement 16	Mode Shape: 16 (Value = 732,772 Hz)	0,0193842 mm, Node: 38799	3132,61 mm, Node: 2315
Displacement 18	Mode Shape: 18 (Value = 872,479 Hz)	0,106837 mm, Node: 57671	2037,22 mm, Node: 924
Displacement 19	Mode Shape: 19 (Value = 889,133 Hz)	0,00868428 mm, Node: 47490	2105,59 mm, Node: 3052
Displacement 20	Mode Shape: 20 (Value = 985,644 Hz)	0,0222579 mm, Node: 35492	3661,65 mm, Node: 2163
Displacement 21	Mode Shape: 21 (Value = 992,086 Hz)	0,164364 mm, Node: 56060	3325,77 mm, Node: 82771
Displacement 22	Mode Shape: 22 (Value = 1018,63 Hz)	0,00942916 mm, Node: 39776	3822,78 mm, Node: 1344
Displacement 23	Mode Shape: 23 (Value = 1242,37 Hz)	0,0454063 mm, Node: 82966	4243,03 mm, Node: 1264
Displacement 25	Mode Shape: 25 (Value = 1335,15 Hz)	0,0303029 mm, Node: 54714	3370,97 mm, Node: 2734
Displacement 27	Mode Shape: 27 (Value = 1465,53 Hz)	0,0194326 mm, Node: 42088	4561,09 mm, Node: 1282
Displacement 28	Mode Shape: 28 (Value = 1514,03 Hz)	0,110482 mm, Node: 57640	2981,67 mm, Node: 67556
Displacement 30	Mode Shape: 30 (Value = 1661,66 Hz)	0,0202945 mm, Node: 38846	3969,34 mm, Node: 2296

6. CONCLUSION

The paper presents the metodics and results of the simulative investigation of the circular saw with compensator grooves. The natural frequencies and mode shapes of the studied circular saw are obtained. The estimation is done by a current application programme, taking into account the typical characteristics in the structure of the kind of circular saws and the physical-mechanical characteristics of their materials. The results of the investigation prove practical significance of the developed mechanic-mathematical model and the methods for study of the circular saw with compensator grooves. They point the possibilities for determinations of resonant regimes and they are basis for their detailed studying.

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